Chapter 8 Systems of Equations

Section 8.1 Solving Systems of Equations Graphically

Section 8.1 Page 435 Question 1

a) System A models the situation: to go off a ramp at different heights means two positive vertical intercepts and in this system the launch angles are different causing the bike with the lower trajectory to land sooner. System B is not correct because it shows both jumps starting from same height. System C has one start from zero, which would mean no ramp. In System D, a steeper trajectory would mean being in the air longer but the rider is going at the same speed.

b) The rider was at the same height and at same time after leaving the jump regardless of which ramp was chosen.

Section 8.1 Page 435 Question 2

For $(0, -5)$:
In $y = -x^2 + 4x - 5$

Left Side	Right Side
$y = -5$	$-x^2 + 4x - 5$
	$= -(0)^2 + 4(0) - 5$
	$= -5$

Left Side = Right Side

In $y = x - 5$

Left Side	Right Side
$y = -5$	$x - 5$
	$= 0 - 5$
	$= -5$

Left Side = Right Side

For $(3, -2)$:
In $y = -x^2 + 4x - 5$

Left Side	Right Side
$y = -2$	$-x^2 + 4x - 5$
	$= -(3)^2 + 4(3) - 5$
	$= -2$

Left Side = Right Side

In $y = x - 5$

Left Side
$y = -2$

Right Side
$x - 5$
$= 3 - 5$
$= -2$

Left Side = Right Side

So, both solutions are verified.

Section 8.1 Page 435 Question 3

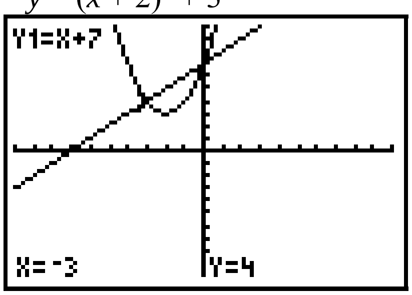
a) The equations $x + y + 3 = 0$ and $x^2 + 6x + y + 7 = 0$ define a linear-quadratic system. The solutions are $(-4, 1)$ and $(-1, -2)$.

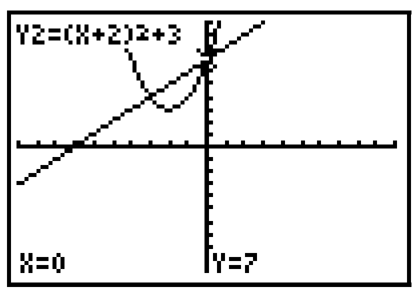
b) The equations $y = x^2 - 4x + 7$ and $y = \frac{1}{2}x^2 - 2x + 3$ define a quadratic-quadratic system. The system has no solution.

c) The equations $y = 2x^2 - 4x - 2$ and $y = -4$ define a linear-quadratic system. The solution is $(1, -4)$.

Section 8.1 Page 435 Question 4

a) $y = x + 7$
$y = (x + 2)^2 + 3$





From the graph, the solutions are $(-3, 4)$ and $(0, 7)$.

For $(-3, 4)$:
In $y = x + 7$

Left Side
$y = 4$

Right Side
$x + 7$
$= -3 + 7$
$= 4$

Left Side = Right Side

In $y = (x + 2)^2 + 3$

Left Side	Right Side
$y = 4$	$(x + 2)^2 + 3$
	$= (-3 + 2)^2 + 3$
	$= 1 + 3$
	$= 4$

Left Side = Right Side

For (0, 7):
In $y = x + 7$

Left Side	Right Side
$y = 7$	$x + 7$
	$= 0 + 7$
	$= 7$

Left Side = Right Side

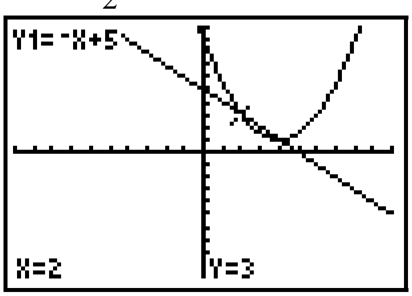
In $y = (x + 2)^2 + 3$

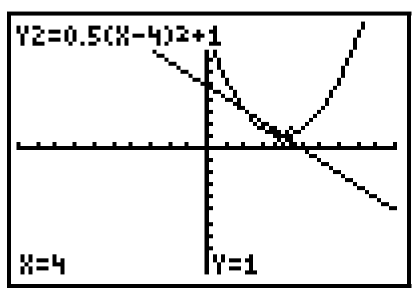
Left Side	Right Side
$y = 7$	$(x + 2)^2 + 3$
	$= (0 + 2)^2 + 3$
	$= 4 + 3$
	$= 7$

Left Side = Right Side

b) $f(x) = -x + 5$

$g(x) = \frac{1}{2}(x - 4)^2 + 1$





From the graph, the solutions are (2, 3) and (4, 1).
For (2, 3):
In $f(x) = -x + 5$

Left Side	Right Side
$f(x) = 3$	$-x + 5$
	$= -2 + 5$
	$= 3$

Left Side = Right Side

In $g(x)=\frac{1}{2}(x-4)^2+1$

Left Side	Right Side
$g(x) = 3$	$\frac{1}{2}(x-4)^2+1$
	$=\frac{1}{2}(2-4)^2+1$
	$=3$

Left Side = Right Side

For (4, 1):

In $f(x) = -x + 5$

Left Side	Right Side
$f(x) = 1$	$-x + 5$
	$= -4 + 5$
	$= 1$

Left Side = Right Side

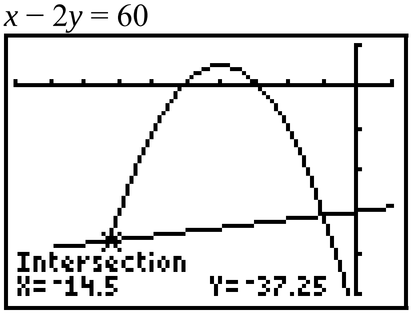
In $g(x)=\frac{1}{2}(x-4)^2+1$

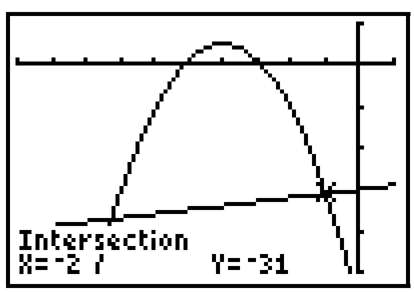
Left Side	Right Side
$g(x) = 1$	$\frac{1}{2}(x-4)^2+1$
	$=\frac{1}{2}(4-4)^2+1$
	$=1$

Left Side = Right Side

c) $x^2 + 16x + y = -59$

$x - 2y = 60$





From the graph, the solutions are (–14.5, –37.25) and (–2, –31).

For (–14.5, –37.25):

In $x^2 + 16x + y = -59$

Left Side	Right Side
$x^2 + 16x + y$	-59
$= (-14.5)^2 + 16(-14.5) + (-37.25)$	
$= 210.25 - 232 - 37.25$	
$= -59$	

Left Side = Right Side

In $x - 2y = 60$

Left Side	Right Side
$x - 2y$	60
$= -14.5 - 2(-37.25)$	
$= 60$	

Left Side = Right Side

For (–2, –31):

In $x^2 + 16x + y = -59$

Left Side	Right Side
$x^2 + 16x + y$	-59
$= (-2)^2 + 16(-2) + (-31)$	
$= 4 - 32 - 31$	
$= -59$	

Left Side = Right Side

In $x - 2y = 60$

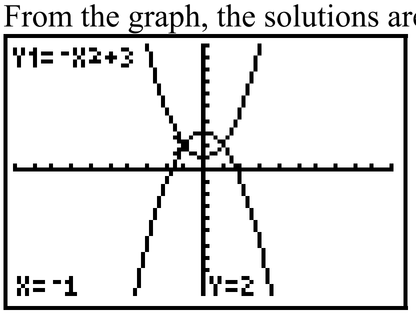
Left Side	Right Side
$x - 2y$	60
$= -2 - 2(-31)$	
$= 60$	

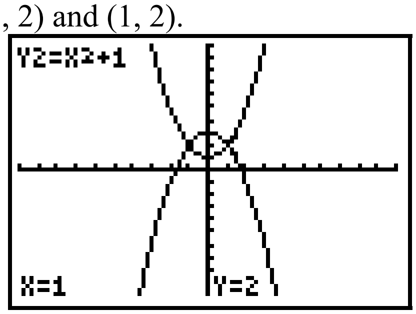
Left Side = Right Side

d) $x^2 + y - 3 = 0$

$x^2 - y + 1 = 0$

From the graph, the solutions are (–1, 2) and (1, 2).





For (–1, 2):
In $x^2 + y - 3 = 0$

Left Side	Right Side
$x^2 + y - 3$	0
$= (-1)^2 + 2 - 3$	
$= 0$	

Left Side = Right Side

In $x^2 - y + 1 = 0$

Left Side	Right Side
$x^2 - y + 1$	0
$= (-1)^2 - 2 + 1$	
$= 0$	

Left Side = Right Side

For (1, 2):
In $x^2 + y - 3 = 0$

Left Side	Right Side
$x^2 + y - 3$	$= 0$
$= (1)^2 + 2 - 3$	
$= 0$	

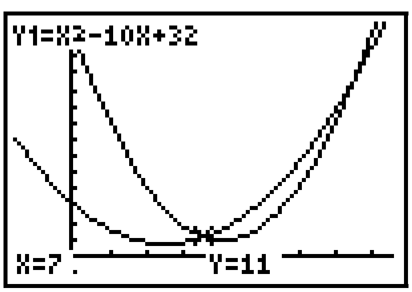
Left Side = Right Side

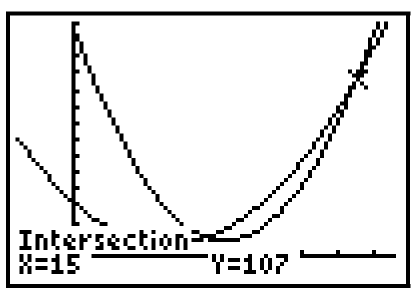
In $x^2 - y + 1 = 0$

Left Side	Right Side
$x^2 - y + 1$	0
$= (1)^2 - 2 + 1$	
$= 0$	

Left Side = Right Side

e) $y = x^2 - 10x + 32$
$y = 2x^2 - 32x + 137$





From the graph, the solutions are (7, 11) and (15, 107).

For (7, 11):
In $y = x^2 - 10x + 32$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 11$	$= 7^2 - 10(7) + 32$
	$= 49 - 70 + 32$
	$= 11$

Left Side = Right Side

In $y = 2x^2 - 32x + 137$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 11$	$= 2(7)^2 - 32(7) + 137$
	$= 98 - 224 + 137$
	$= 11$

Left Side = Right Side

For (15, 107):
In $y = x^2 - 10x + 32$

Left Side	Right Side
y	$x^2 - 10x + 32$
$= 107$	$= 15^2 - 10(15) + 32$
	$= 225 - 150 + 32$
	$= 107$

Left Side = Right Side

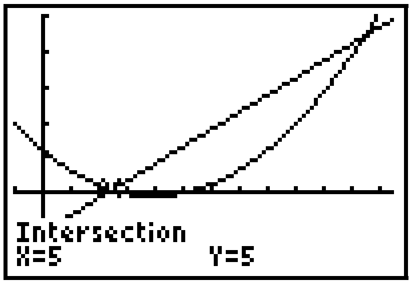
In $y = 2x^2 - 32x + 137$

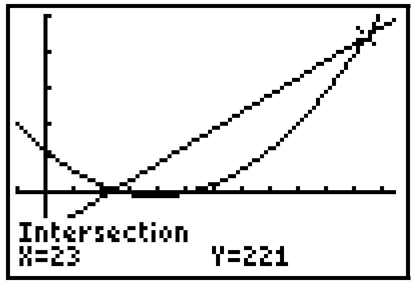
Left Side	Right Side
y	$2x^2 - 32x + 137$
$= 107$	$= 2(15)^2 - 32(15) + 137$
	$= 450 - 480 + 137$
	$= 107$

Left Side = Right Side

Section 8.1 Page 436 Question 5

a) $h = d^2 - 16d + 60$
$h = 12d - 55$





From the graph, the solutions are (5, 5) and (23, 221).

For (5, 5):
In $h = d^2 - 16d + 60$

Left Side	Right Side
h	$d^2 - 16d + 60$
$= 5$	$= 5^2 - 16(5) + 60$
	$= 25 - 80 + 60$
	$= 5$

Left Side = Right Side

In $h = 12d - 55$

Left Side	Right Side
h	$12d - 55$
$= 5$	$= 12(5) - 55$
	$= 5$

Left Side = Right Side

For (23, 221):
In $h = d^2 - 16d + 60$

Left Side	Right Side
h	$d^2 - 16d + 60$
$= 221$	$= 23^2 - 16(23) + 60$
	$= 529 - 368 + 60$
	$= 221$

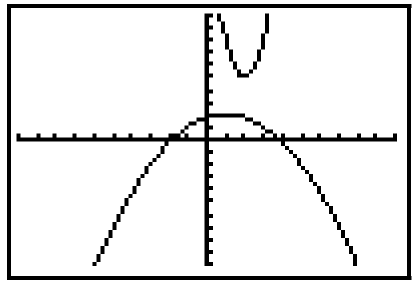
Left Side = Right Side

In $h = 12d - 55$

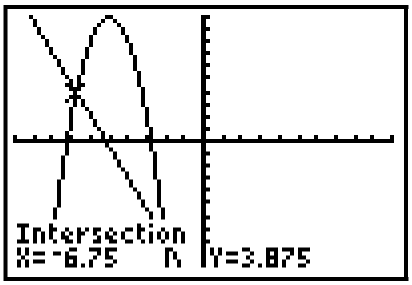
Left Side	Right Side
h	$12d - 55$
$= 221$	$= 12(23) - 55$
	$= 221$

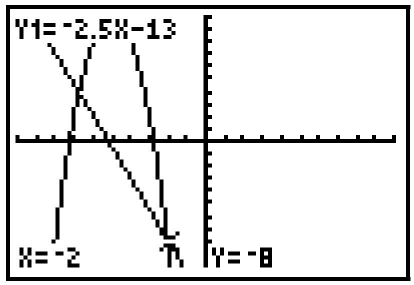
Left Side = Right Side

b) $p = 3q^2 - 12q + 17$
$p = -0.25q^2 + 0.5q + 1.75$
From the graph, there are no solutions.



c) $2v^2 + 20v + t = -40$
$5v + 2t + 26 = 0$





From the graph, the solutions are (–6.75, 3.875) and (–2, –8).

For (–6.75, 3.875):
In $2v^2 + 20v + t = -40$

Left Side	Right Side
$2v^2 + 20v + t$	-40
$= 2(-6.75)^2 + 20(-6.75) + 3.875$	
$= 91.125 - 135 + 3.875$	
$= -40$	

Left Side = Right Side

In $5v + 2t + 26 = 0$

Left Side	Right Side
$5v + 2t + 26$	0
$= 5(-6.75) + 2(3.875) + 26$	
$= 0$	

Left Side = Right Side

For (–2, –8):
In $2v^2 + 20v + t = -40$

Left Side	Right Side
$2v^2 + 20v + t$	-40
$= 2(-2)^2 + 20(-2) + (-8)$	
$= 8 - 40 + 8$	
$= -40$	

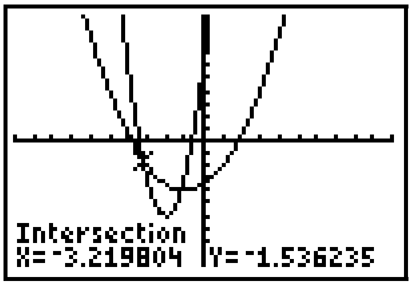
Left Side = Right Side

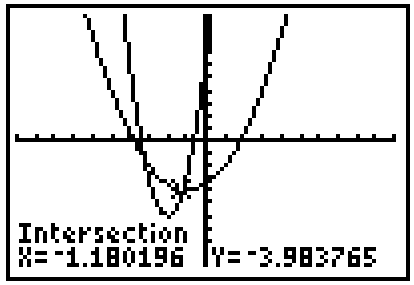
In $5v + 2t + 26 = 0$

Left Side | Right Side
$5v + 2t + 26$ | 0
$= 5(-2) + 2(-8) + 26$
$= 0$

Left Side = Right Side

d) $n^2 + 2n - 2m - 7 = 0$
$3n^2 + 12n - m + 6 = 0$





From the graph, the solutions are approximately (–3.22, –1.54) and (–1.18, –3.98).

For (–3.22, –1.54):
In $n^2 + 2n - 2m - 7 = 0$

Left Side | Right Side
$n^2 + 2n - 2m - 7$ | 0
$= (-3.22)^2 + 2(-3.22) - 2(-1.54) - 7$
$= 0.0084$
≈ 0

Left Side = Right Side

In $3n^2 + 12n - m + 6 = 0$

Left Side | Right Side
$3n^2 + 12n - m + 6$ | 0
$= 3(-3.22)^2 + 12(-3.22) - (-1.54) + 6$
$= 0.0052$
≈ 0

Left Side = Right Side

For (–1.18, –3.98):
In $n^2 + 2n - 2m - 7 = 0$

Left Side | Right Side
$n^2 + 2n - 2m - 7$ | 0
$= (-1.18)^2 + 2(-1.18) - 2(-3.98) - 7$
$= -0.0076$
≈ 0

Left Side = Right Side

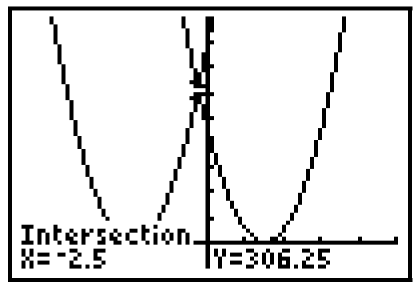
In $3n^2 + 12n - m + 6 = 0$

Left Side
$3n^2 + 12n - m + 6$
$= 3(-1.18)^2 + 12(-1.18) - (-3.98) + 6$
$= -0.0028$
≈ 0

Right Side
0

Left Side = Right Side

e)
$0 = t^2 + 40t - h + 400$
$t^2 = h + 30t - 225$
From the graph, the solution is $(-2.5, 306.25)$.



For $(-2.5, 306.25)$:
In $0 = t^2 + 40t - h + 400$

Left Side
0

Right Side
$t^2 + 40t - h + 400$
$= (-2.5)^2 + 40(-2.5) - 306.25 + 400$
$= 0$

Left Side = Right Side

In $t^2 = h + 30t - 225$

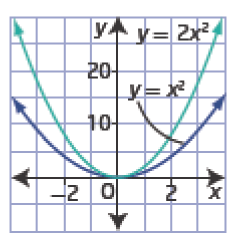
Left Side
t^2
$= (-2.5)^2$
$= 6.25$

Right Side
$h + 30t - 225$
$= 306.25 + 30(-2.5) - 225$
$= 6.25$

Left Side = Right Side

Section 8.1 Page 436 Question 6

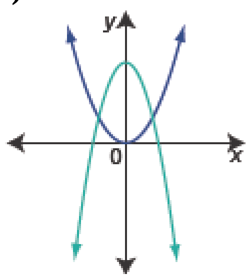
The two parabolas have the same vertex, but different *a* values.
Example: $y = x^2$ and $y = 2x^2$.



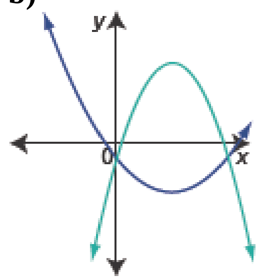
Section 8.1 Page 436 Question 7

Examples:

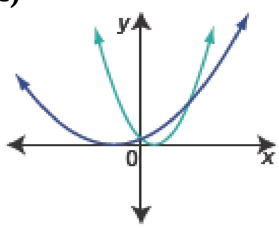
a)



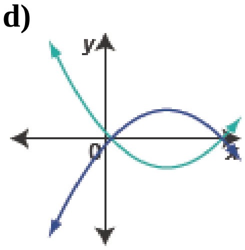
b)



c)



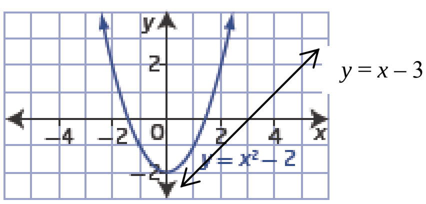
d)



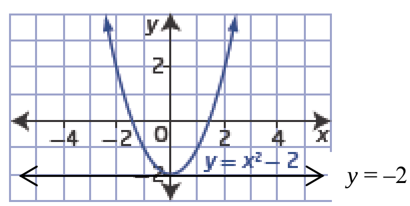
Section 8.1 Page 436 Question 8

Examples:

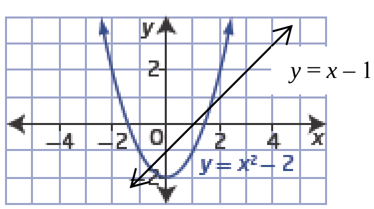
a) $y = x - 3$



b) $y = -2$



c) $y = x - 1$



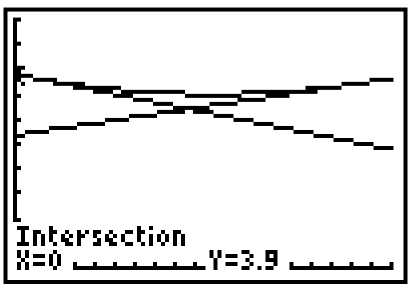
Section 8.1 Page 436 Question 9

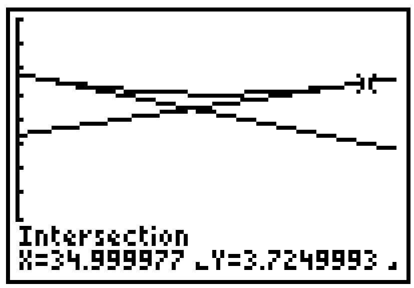
a) The points of intersection on the graph are (100, 3800) and (1000, 8000), to the nearest hundred.

b) When he makes and sells either 100 or 1000 shirts, Jonas makes no profit as costs equal revenue. When he makes more than 100 shirts but less than 1000 he will make a profit.

c) Estimate when the vertical distance between the Cost line and the Revenue parabola is greatest: this is the greatest profit. From the graph it is about 550 shirts, which gives a profit of about $15 500.

Section 8.1 Page 436 Question 10



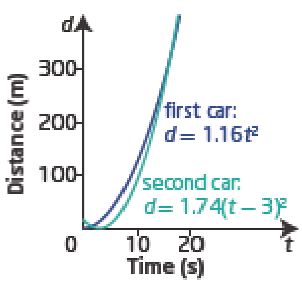


The coordinates of the beginning of the curve are (0, 3.9). The coordinates of the end of the curve are (35, 3.725).

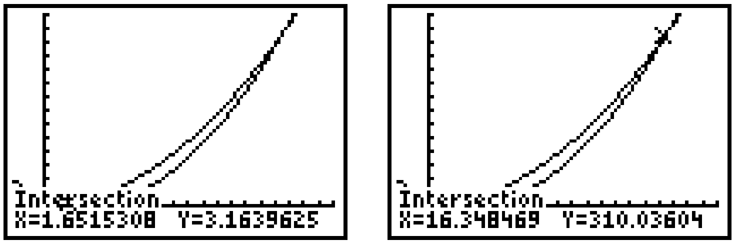
Section 8.1 Page 437 Question 11

a) The system of equations that represents the distance travelled by the two cars is:
$d = 1.16t^2$ and $d = 1.74(t - 3)^2$.

b) Since the first car accelerates for 20 s, and the next car starts 3 s later, a suitable domain for consideration is $0 \leq t \leq 23$.



c)

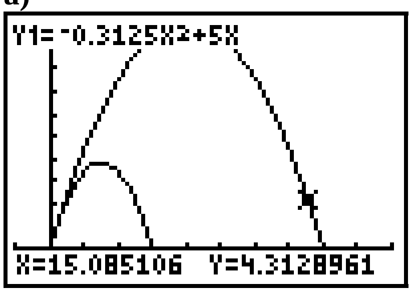


Graphically the two solutions are (1.65, 3.16) and (16.35, 310.04). However, the first is not a solution for the distance between the cars during the test, because at $t = 1.65$ the second car has not yet started.

d) The solution (16.35, 310.04) means that at 16.35 s after the first car starts, both cars have travelled the same distance, about 310.04 m.

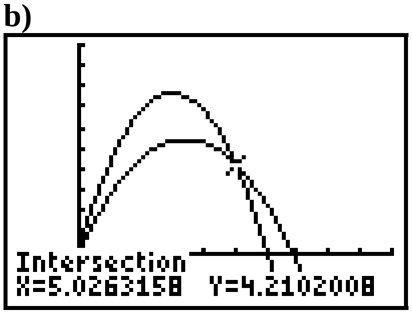
Section 8.1 Page 437 Question 12

a)



Both streams of water start at (0, 0), at the fountain, but they have no other point in common. The tallest stream reaches higher and further from the fountain than the smaller stream.

b)

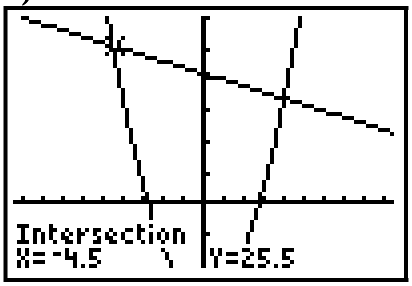


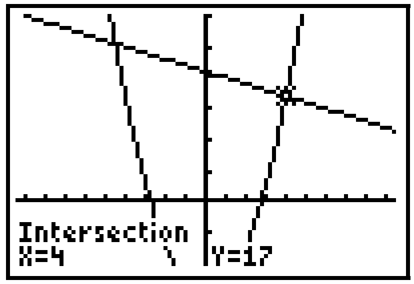
Both streams of water start at (0, 0), but the second stream passes through the other fountain's spray 5.03 m from the fountain, at a height of 4.21 m.

Section 8.1 Page 437 Question 13

a) Let x represent the smaller integer and y the larger integer.
$x + y = 21$
$2x^2 - 15 = y$

b)





One point of intersection does not give integers. The two integers are 4 and 17.

c) For $x + y = 21$:

Left Side $x + y$
$= 4 + 17$
$= 21$

Right Side 21

Left Side = Right Side

For $2x^2 - 15 = y$

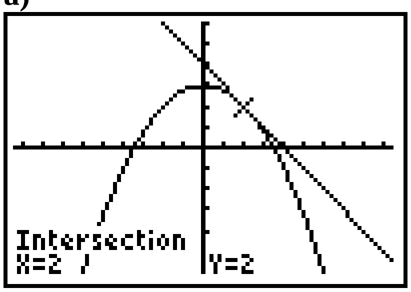
Left Side $2x^2 - 15$
$= 2(4)^2 - 15$
$= 17$

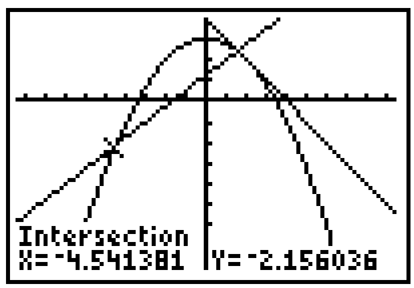
Right Side y
$= 17$

Left Side = Right Side

Section 8.1 Page 438 Question 14

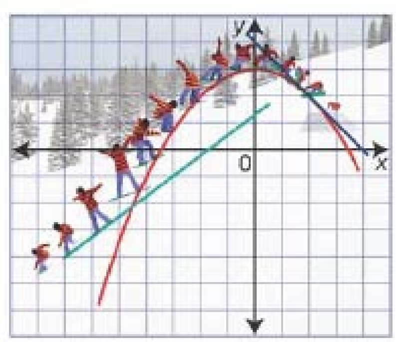
a)





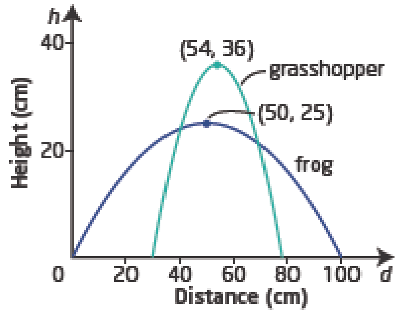
The blue line and parabola intersect at (2, 2). The green line and the parabola intersect at about (−4.54, −2.16).

b) When a snowboarder leaves the jump at a point (2, 2), he lands at a point with the relative location (−4.54, −2.16).



Section 8.1 Page 438 Question 15

a)



b) For the frog:
Starts at (0, 0), lands at (100, 0), maximum at (50, 25).
Using the vertex form of the quadratic equation, the equation is
$y = a(x - 50)^2 + 25$
To determine a, substitute (0, 0).

$$0 = a(0 - 50)^2 + 25$$
$$-25 = 2500a$$
$$a = -0.01$$

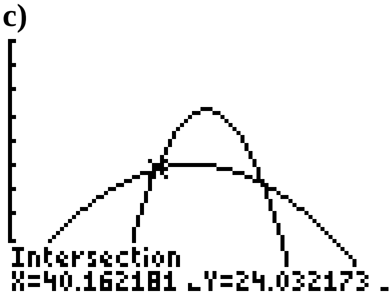
A quadratic equation that models the path of the frog is $y = -0.01(x - 50)^2 + 25$.

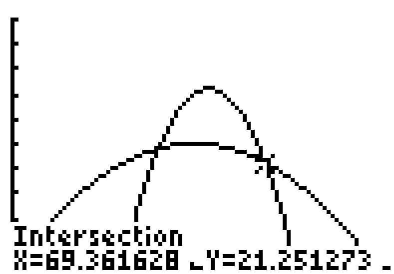
For the grasshopper:
Starts at (30, 0), lands at (78, 0), maximum at (54, 36).
Using the vertex form of the quadratic equation, the equation is
$y = a(x - 54)^2 + 36$
To determine a, substitute (30, 0).

$$0 = a(30 - 54)^2 + 36$$
$$-36 = 576a$$
$$a = -0.0625$$

A quadratic equation that models the path of the frog is $y = -0.0625(x - 54)^2 + 36$.

c)





The solutions for the system of equations $y = -0.01(x - 50)^2 + 25$ and $y = -0.0625(x - 54)^2 + 36$ are (40.16, 24.03) and (69.36, 21.25).

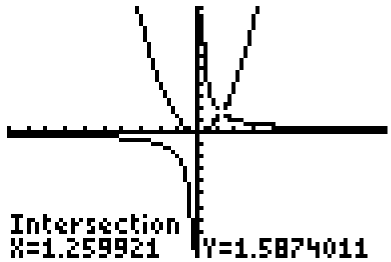
d) These are the locations where the frog and grasshopper are at the same distance and height relative to the frog's starting point. If the frog does not catch the grasshopper at the first point, there is another opportunity. However, we do not know anything about time, *i.e.*, the speed of either one, so the grasshopper may be gone.

Section 8.1 Page 438 Question 16

a) From $\frac{1}{x} = \frac{x}{y} = \frac{y}{2}$ form two equations.

$x^2 = y$ and $y = \frac{2}{x}$.

Use a graph to find their point of intersection.



The solution is approximately (1.26, 1.59).

b) According to the given information, a is the side length and x is the length that gives double the volume. So, when $a = 1$, $x = 1.26$.
A cube with side length 1.26 cm has double the volume of a cube with side length 1 cm.

c) $V = lwh$

$V = 1.26 \times 1.26 \times 1.26$

$V = 2.000\ 376$

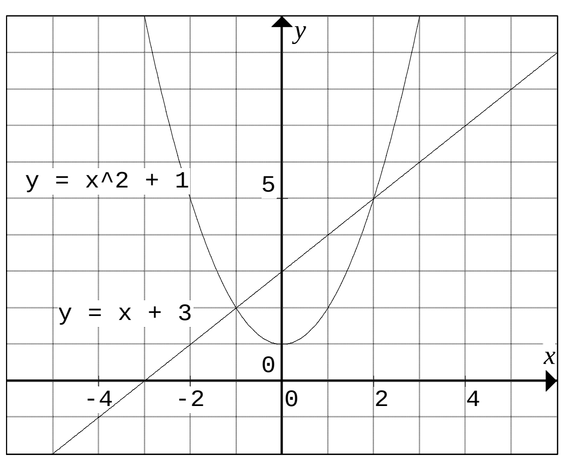
So, the volume of a cube with side length 1.26 cm is very close to 2 cm^3.

d) If x represents the length of one side, then $V = x^3$. For a volume of 2 cm^3, $2 = x^3$. Then, $x = \sqrt[3]{2}$ or approximately 1.26. Menaechmus did not have a calculator to find roots.

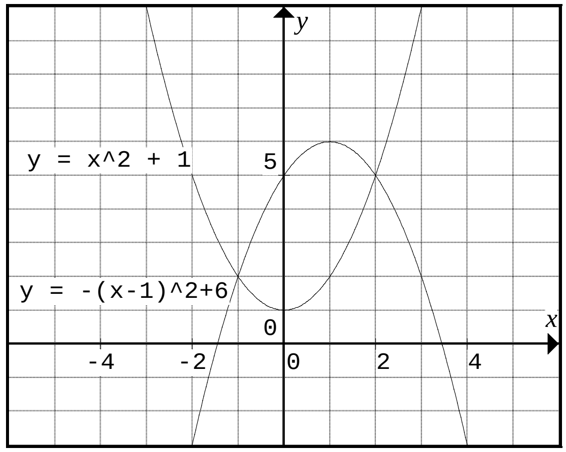
Section 8.1 Page 439 Question 17

Examples:

a) $y = x^2 + 1$ and $y = x + 3$ intersect at (–1, 2) and (2, 5).



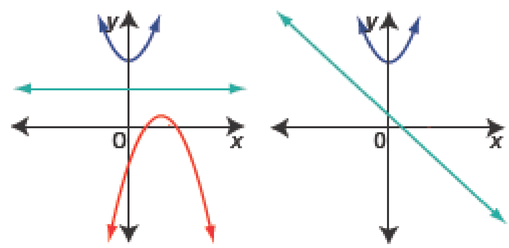
b) $y = x^2 + 1$ and
$y = -(x - 1)^2 + 6$
intersect at (–1, 2) and (2, 5).



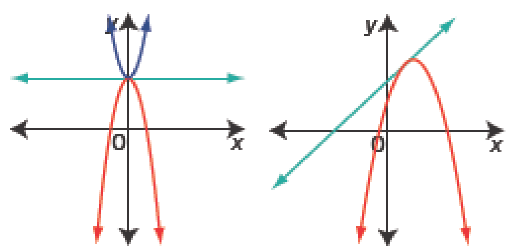
c) $y = x^2 + 1$ and $y = (x – 1)^2 + 2x$ are equivalent equations: the second equation simplifies to $y = x^2 + 1$, so they have infinitely many solutions including (–1, 2) and (2, 5).

Section 8.1 Page 439 Question 18

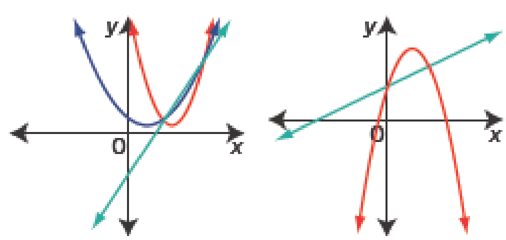
No solution: Two parabolas do not intersect and the line is between them, intersecting neither, or the parabolas are coincident and the line does not interest them.



One solution: two parabolas intersecting once, with a line tangent to both curves, or the parabolas are coincident and the line is tangent.



Two solutions: Two parabolas intersecting twice, with a line passing through both points of intersection, or the parabolas are coincident and the line passes through two points on them.



Section 8.1 Page 439 Question 19

Example: Similarities: a different number of solutions are possible. A linear system can have one solution, no solution, or an infinite number of solutions. A linear-quadratic system has the same possibilities but it may also have two possible solutions. Both systems can be solved graphically or algebraically and checked by substituting.
Differences: Some systems involving quadratic equations cannot be solved by elimination. The systems involving quadratic equations are more difficult to solve.

Section 8.1 Page 439 Question 20

a) Two solutions. The y-intercept of the line $y = x + 1$ is above the vertex (0, 0) of the parabola $y = x^2$, and parabola opens upward.

b) The system defined by the equations $y = 2x^2 + 3$ and $y = -2x - 5$ has no solution. The parabola's vertex is at (0, 3) and it opens upward, while the line has y-intercept −5 and it has negative slope so it lies entirely below the parabola.

c) The system defined by the equations $y = (x - 4)^2 + 1$ and $y = \frac{1}{3}(x-4)^2 + 2$ has two solutions. One vertex is directly above the other but the upper parabola, $y = \frac{1}{3}(x-4)^2 + 2$, has a smaller vertical stretch factor so it will intersect the other parabola in two places.

d) The system defined by the equations $y = 2(x + 8)^2 - 9$ and $y = -2(x + 8)^2 - 9$ has one solution at their common vertex (–8, –9). One opens upward, the other downward.

e) The system defined by the equations $y = 2(x - 3)^2 + 1$ and $y = -2(x - 3)^2 - 1$ have no solution. The first parabola has its vertex at (3, 1) and opens upward. The second parabola has it vertex at (3, −1) and opens downward.

f) The system defined by the equations $y = (x + 5)^2 - 1$ and $y = x^2 + 10x + 24$ has an infinite number of solutions. When the first equation is expanded and simplified, it is exactly the same as the second equation.

Section 8.2 Solving Systems of Equations Algebraically

Section 8.2 Page 451 Question 1

In $k + p = 12$:
Left Side
$k + p$
$= 5 + 7$
$= 12$
$=$ Right Side

In $4k^2 - 2p = 86$:
Left Side
$4k^2 - 2p$
$= 4(5)^2 - 2(7)$
$= 86$
$=$ Right Side

So, (5, 7) is a solution.

Section 8.2 Page 451 Question 2

In $18w^2 - 16z^2 = -7$:

$$\begin{aligned}\text{Left Side} &= 18\left(\frac{1}{3}\right)^2 - 16\left(\frac{3}{4}\right)^2\\ &= 18\left(\frac{1}{9}\right) - 16\left(\frac{9}{16}\right)\\ &= 2 - 9\\ &= -7\\ &= \text{Right Side}\end{aligned}$$

In $144w^2 + 48z^2 = 43$:

$$\begin{aligned}\text{Left Side} &= 144\left(\frac{1}{3}\right)^2 + 48\left(\frac{3}{4}\right)^2\\ &= 16 + 27\\ &= 43\\ &= \text{Right Side}\end{aligned}$$

So, $\left(\frac{1}{3}, \frac{3}{4}\right)$ is a solution.

Section 8.2 Page 451 Question 3

a) $x^2 - y + 2 = 0$ ①
$4x = 14 - y$ ②
From ②, $y = 14 - 4x$. Substitute into ①.
$x^2 - (14 - 4x) + 2 = 0$
$x^2 - 14 + 4x + 2 = 0$
$x^2 + 4x - 12 = 0$
$(x + 6)(x - 2) = 0$
$x = -6$ or $x = 2$
Substitute into ②.

When $x = -6$:
$4x = 14 - y$
$4(-6) = 14 - y$
$y = 14 + 24$
$y = 38$

When $x = 2$:
$4x = 14 - y$
$4(2) = 14 - y$
$y = 14 - 8$
$y = 6$

Verify (–6, 38).

In $x^2 - y + 2 = 0$:
Left Side = $(–6)^2 – 38 + 2$
= 36 – 38 + 2
= 0
= Right Side

In $4x = 14 – y$:
Left Side = $4(–6)$ = –24
Right Side = $14 – 38$ = –24
Left Side = Right Side

Verify (2, 6).
In $x^2 - y + 2 = 0$:
Left Side = $2^2 – 6 + 2$
= 0
= Right Side

In $4x = 14 – y$:
Left Side = $4(2)$ = 8
Right Side = $14 – 6$ = 8
Left Side = Right Side

Both solutions, (–6, 38) and (2, 6), check.

b) $2x^2 - 4x + y = 3$ ①
$4x - 2y = -7$ ②
From ②, $y = 2x + 3.5$. Substitute into ①.
$2x^2 - 4x + 2x + 3.5 = 3$
$2x^2 - 2x + 0.5 = 0$
$4x^2 – 4x + 1 = 0$
$(2x – 1)(2x – 1) = 0$
$x = 0.5$
Substitute into ②.
$4x - 2y = -7$
$4(0.5) – 2y = –7$
$2 + 7 = 2y$
$y = 4.5$

Verify (0.5, 4.5).
In $2x^2 - 4x + y = 3$:
Left Side = $2(0.5)^2 – 4(0.5) + 4.5$
= 3
= Right Side

In $4x - 2y = -7$:
Left Side = $4(0.5) – 2(4.5)$
= –7
= Right Side

The solution (0.5, 4.5) checks.

c) $7d^2 + 5d - t - 8 = 0$ ①
$10d - 2t = -40$ ②
From ②, $t = 5d + 20$. Substitute into ①.
$7d^2 + 5d - (5d + 20) – 8 = 0$
$7d^2 – 28 = 0$
$7(d – 2)(d + 2) = 0$
$d = 2$ or $d = –2$
Substitute into ②.

When $d = 2$:
$10d - 2t = -40$
$10(2) – 2t = –40$

When $d = –2$:
$10d - 2t = -40$
$10(–2) – 2t = –40$

$60 = 2t$ $\quad\quad\quad\quad$ $-2t = -20$

$t = 30$ $\quad\quad\quad\quad$ $t = 10$

Verify (2, 30).

In $7d^2 + 5d - t - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 7(2)^2 + 5(2) - 30 - 8 \\ &= 28 + 10 - 38 \\ &= 0 \\ &= \text{Right Side}\end{aligned}$$

In $10d - 2t = -40$:

$$\begin{aligned}\text{Left Side} &= 10(2) - 2(30) \\ &= 20 - 60 \\ &= -40 \\ &= \text{Right Side}\end{aligned}$$

Verify (–2, 10).

In $7d^2 + 5d - t - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 7(-2)^2 + 5(-2) - 10 - 8 \\ &= 28 - 10 - 18 \\ &= 0 \\ &= \text{Right Side}\end{aligned}$$

In $10d - 2t = -40$:

$$\begin{aligned}\text{Left Side} &= 10(-2) - 2(10) \\ &= -20 - 20 \\ &= -40 \\ &= \text{Right Side}\end{aligned}$$

Both solutions, (2, 30) and (–2, 10), check.

d) $3x^2 + 4x - y - 8 = 0$ ①

$y + 3 = 2x^2 + 4x$ ②

From ②, $y = 2x^2 + 4x - 3$. Substitute in ①.

$3x^2 + 4x - (2x^2 + 4x - 3) - 8 = 0$

$x^2 - 5 = 0$

$x = \pm\sqrt{5}$

$x \approx 2.24$ or $x \approx -2.24$

Substitute in ②.

When $x = \sqrt{5}$:

$y + 3 = 2x^2 + 4x$

$y + 3 = 2(\sqrt{5})^2 + 4(\sqrt{5})$

$y = 7 + 4\sqrt{5}$

$y \approx 15.94$

When $x = -\sqrt{5}$:

$y + 3 = 2x^2 + 4x$

$y + 3 = 2(-\sqrt{5})^2 + 4(-\sqrt{5})$

$y = 7 - 4\sqrt{5}$

$y \approx -1.94$

Verify $(\sqrt{5}, 7 + 4\sqrt{5})$.

In $3x^2 + 4x - y - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 3(\sqrt{5})^2 + 4(\sqrt{5}) - (7 + 4\sqrt{5}) - 8 \\ &= 15 - 15 \\ &= 0 \\ &= \text{Right Side}\end{aligned}$$

In $y + 3 = 2x^2 + 4x$:

Left Side	Right Side
$= 7 + 4\sqrt{5} + 3$	$= 2(\sqrt{5})^2 + 4(\sqrt{5})$
$= 10 + 4\sqrt{5}$	$= 10 + 4\sqrt{5}$

Left Side = Right Side

Verify $(-\sqrt{5}, 7 - 4\sqrt{5})$.

In $3x^2 + 4x - y - 8 = 0$:

$$\begin{aligned}\text{Left Side} &= 3(-\sqrt{5})^2 + 4(-\sqrt{5}) - (7 - 4\sqrt{5}) - 8 \\ &= 15 - 15 \\ &= 0\end{aligned}$$

In $y + 3 = 2x^2 + 4x$:

Left Side	Right Side
$= 7 - 4\sqrt{5} + 3$	$= 2(-\sqrt{5})^2 + 4(-\sqrt{5})$
$= 10 - 4\sqrt{5}$	$= 10 - 4\sqrt{5}$

= Right Side　　　　Left Side = Right Side

The two solutions check. The approximate answers are (2.24, 15.94) and (–2.24, –1.94).

e) $y + 2x = x^2 - 6$ ①
$x + y - 3 = 2x^2$ ②
From ②, $y = 2x^2 - x + 3$. Substitute in ①.
$2x^2 - x + 3 + 2x = x^2 - 6$
$x^2 + x + 9 = 0$
Use the quadratic formula with $a = 1$, $b = 1$, and $c = 9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{-23}}{4}$$

This system has no real solutions.

Section 8.2　Page 452　Question 4

a) $6x^2 - 3x = 2y - 5$ ①
$2x^2 + x = y - 4$ ②
Multiply ② by –2.
$6x^2 - 3x = 2y - 5$ ①
$-4x^2 - 2x = -2y + 8$ ③
Add ① + ③.
$2x^2 - 5x = 3$
$2x^2 - 5x - 3 = 0$
$(2x + 1)(x - 3) = 0$
$x = -0.5$ or $x = 3$
Substitute in ②.

When $x = -0.5$:
$2x^2 + x = y - 4$
$2(0.5)^2 + (-0.5) + 4 = y$
$y = 4$

When $x = 3$:
$2x^2 + x = y - 4$
$2(3)^2 + 3 + 4 = y$
$y = 25$

Verify (–0.5, 4).
In ①:
Left Side $= 6x^2 - 3x$
$= 6(-0.5)^2 - 3(-0.5)$
$= 3$

Right Side $= 2y - 5$
$= 2(4) - 5$
$= 3$

Left Side = Right Side

In ②:

Left Side $= 2x^2 + x$ | Right Side $= y - 4$

$= 2(-0.5)^2 + (-0.5)$ | $= 4 - 4$

$= 0$ | $= 0$

Left Side = Right Side

Verify (3, 25).

In ①:

Left Side $= 6x^2 - 3x$ | Right Side $= 2y - 5$

$= 6(3)^2 - 3(3)$ | $= 2(25) - 5$

$= 45$ | $= 45$

Left Side = Right Side

In ②:

Left Side $= 2x^2 + x$ | Right Side $= y - 4$

$= 2(3)^2 + 3$ | $= 25 - 4$

$= 21$ | $= 21$

Left Side = Right Side

Both solutions, (–0.5, 4) and (3, 25), check.

b) $x^2 + y = 8x + 19$ ①

$x^2 - y = 7x - 11$ ②

Add ① + ②.

$2x^2 = 15x + 8$

$2x^2 - 15x - 8 = 0$

$(2x + 1)(x - 8) = 0$

$x = -0.5$ or $x = 8$

Substitute in ①.

When $x = -0.5$:

$y = 8x + 19 - x^2$

$y = 8(-0.5) + 19 - (-0.5)^2$

$y = 14.75$

When $x = 8$:

$y = 8x + 19 - x^2$

$y = 8(8) + 19 - 8^2$

$y = 19$

Verify (–0.5, 14.75).

In ①:

Left Side $= x^2 + y$ | Right Side $= 8x + 19$

$= (-0.5)^2 + 14.75$ | $= 8(-0.5) + 19$

$= 15$ | $= 15$

Left Side = Right Side

In ②:

Left Side $= x^2 - y$ | Right Side $= 7x - 11$

$= (-0.5)^2 + 14.75$ | $= 7(-0.5) - 11$

$= -14.5$ | $= -14.5$

Left Side = Right Side

Verify (8, 19).
In ①:

Left Side $= x^2 + y$
$= 8^2 + 19$
$= 83$

Right Side $= 8x + 19$
$= 8(8) + 19$
$= 83$

Left Side = Right Side

In ②:

Left Side $= x^2 - y$
$= 8^2 - 19$
$= 45$

Right Side $= 7x - 11$
$= 7(8) - 11$
$= 45$

Left Side = Right Side

Both solutions, (–0.5, 14.75) and (8, 19), check.

c) $2p^2 = 4p - 2m + 6$ ①
$5m + 8 = 10p + 5p^2$ ②
Multiply ① by 5 and ② by –2, and rearrange terms.
$10p^2 - 20p - 30 = -10m$ ③
$10p^2 + 20p - 16 = 10m$ ④
Add ③ + ④.
$20p^2 - 46 = 0$
$p = \pm\sqrt{2.3}$
$p \approx 1.52$ or $p \approx -1.52$
Substitute in ②.

When $p = \sqrt{2.3}$:
$5m = 10p + 5p^2 - 8$
$5m = 10(\sqrt{2.3}) + 5(\sqrt{2.3})^2 - 8$
$5m = 10\sqrt{2.3} + 11.5 - 8$
$m = 2\sqrt{2.3} + 0.7$
$m \approx 3.73$

When $p = -\sqrt{2.3}$:
$5m = 10p + 5p^2 - 8$
$5m = 10(-\sqrt{2.3}) + 5(-\sqrt{2.3})^2 - 8$
$5m = -10\sqrt{2.3} + 3.5$
$m = -2\sqrt{2.3} + 0.7$
$m \approx -2.33$

Verify ($\sqrt{2.3}$, $2\sqrt{2.3} + 0.7$).
In ①:

Left Side $= 2p^2$
$= 2(\sqrt{2.3})^2$
$= 4.6$

Right Side $= 4p - 2m + 6$
$= 4(\sqrt{2.3}) - 2(2\sqrt{2.3} + 0.7) + 6$
$= 4.6$

Left Side = Right Side

In ②:

Left Side $= 5m + 8$
$= 5(2\sqrt{2.3} + 0.7) + 8$
$= 10\sqrt{2.3} + 3.5 + 8$
$= 10\sqrt{2.3} + 11.5$

Right Side $= 10p + 5p^2$
$= 10(\sqrt{2.3}) + 5(\sqrt{2.3})^2$
$= 10\sqrt{2.3} + 11.5$

Left Side = Right Side

Verify $(-\sqrt{2.3}, -2\sqrt{2.3} + 0.7)$.

In ①:

Left Side $= 2p^2$
$= 2(-\sqrt{2.3})^2$
$= 4.6$

Right Side $= 4p - 2m + 6$
$= 4(-\sqrt{2.3}) - 2(-2\sqrt{2.3} + 0.7) + 6$
$= 4.6$

Left Side = Right Side

In ②:

Left Side $= 5m + 8$
$= 5(-2\sqrt{2.3} + 0.7) + 8$
$= -10\sqrt{2.3} + 3.5 + 8$
$= -10\sqrt{2.3} + 11.5$

Right Side $= 10p + 5p^2$
$= 10(-\sqrt{2.3}) + 5(-\sqrt{2.3})^2$
$= -10\sqrt{2.3} + 11.5$

Left Side = Right Side

Both solutions, $(\sqrt{2.3}, 2\sqrt{2.3} + 0.7)$ and $(-\sqrt{2.3}, -2\sqrt{2.3} + 0.7)$, check. The solutions are approximately (1.52, 3.73) and (–1.52, –2.33).

d) $9w^2 + 8k = -14$ ①
$w^2 + k = -2$ ②

Multiply ② by –8.

$-8w^2 - 8k = 16$ ③

Add ① and ③.

$w^2 = 2$

$w = \pm\sqrt{2}$

Substitute in ②.

When $w = \sqrt{2}$:
$w^2 + k = -2$
$(\sqrt{2})^2 + k = -2$
$k = -4$

When $w = -\sqrt{2}$:
$w^2 + k = -2$
$(-\sqrt{2})^2 + k = -2$
$k = -4$

Verify $(\sqrt{2}, -4)$.

In ①:

Left Side $= 9w^2 + 8k$
$= 9(\sqrt{2})^2 + 8(-4)$
$= -14$
= Right Side

In ②:

Left Side $= w^2 + k$
$= (\sqrt{2})^2 + (-4)$
$= -2$
= Right Side

Verify $(-\sqrt{2}, -4)$.

In ①:

Left Side $= 9w^2 + 8k$
$= 9(-\sqrt{2})^2 + 8(-4)$
$= -14$
= Right Side

In ②:

Left Side $= w^2 + k$
$= (-\sqrt{2})^2 + (-4)$
$= -2$
= Right Side

Both solutions, $(\sqrt{2}, -4)$ and $(-\sqrt{2}, -4)$, check. The solutions are approximately (1.41, –4) and (–1.41, –4).

e) $4h^2 - 8t = 6$ ①
$6h^2 - 9 = 12t$ ②
Multiply ① by 3 and ② by 2. Align like terms.
$12h^2 - 24t = 18$ ③
$12h^2 - 24t = 18$ ④
Both original equations are equivalent. They have an infinite number of solutions.

Section 8.2 Page 452 Question 5

a) $y - 1 = -\frac{7}{8}x$ ①
$3x^2 + y = 8x - 1$ ②
I will use the substitution method because I can easily obtain an expression for y from equation ①.
From ① $y = 1 - \frac{7}{8}x$. Substitute in ②.
$$3x^2 + 1 - \frac{7}{8}x = 8x - 1$$
$$24x^2 + 16 - 7x - 64x = 0$$
$$24x^2 - 71x + 16 = 0$$
Use the quadratic formula with $a = 24$, $b = -71$, and $c = 16$.
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-71) \pm \sqrt{(-71)^2 - 4(24)(16)}}{2(24)}$$
$$x = \frac{71 \pm \sqrt{3505}}{48}$$
$x \approx 2.71$ or $x \approx 0.25$

Substitute in ①.

When $x = \frac{71 + \sqrt{3505}}{48}$:
$$y = 1 - \frac{7}{8}\left(\frac{71 + \sqrt{3505}}{48}\right)$$
$$y = \frac{-113 - 7\sqrt{3505}}{384}$$
$y \approx -1.37$

When $x = \frac{71 - \sqrt{3505}}{48}$:
$$y = 1 - \frac{7}{8}\left(\frac{71 - \sqrt{3505}}{48}\right)$$
$$y = \frac{-113 + 7\sqrt{3505}}{384}$$
$y \approx 0.78$

The solutions are approximately (2.71, –1.37) and (0.25, 0.78).

b) $8x^2 + 5y = 100$ ①
$6x^2 - x - 3y = 5$ ②
I will use the elimination method because I can eliminate y by adding, if I first multiply ① by 3 and ② by 5.

$24x^2 + 15y = 300$ ③
$30x^2 - 5x - 15y = 25$ ④
Add ③ and ④.
$54x^2 - 5x = 325$
$54x^2 - 5x - 325 = 0$
Use the quadratic formula with $a = 54$, $b = -5$, and $c = 325$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(54)(-325)}}{2(54)}$$

$$x = \frac{5 \pm \sqrt{70\ 225}}{108}$$

$$x = \frac{5 \pm 265}{108}$$

$x = 2.5$ or $x \approx -2.41$

Substitute in ① to determine y.

When $x = 2.5$:

$8(2.5)^2 + 5y = 100$
$5y = 100 - 50$
$y = 10$

When $x = -\dfrac{260}{108}$:

$$8\left(-\frac{260}{108}\right)^2 + 5y = 100$$
$$5y = 53.635...$$
$$y \approx 10.73$$

The solutions are (2.5, 10) and approximately (–2.41, 10.73).

c) $x^2 - \frac{48}{9}x + \frac{1}{3}y + \frac{1}{3} = 0$ ①

$-\frac{5}{4}x^2 - \frac{3}{2}x + \frac{1}{4}y - \frac{1}{2} = 0$ ②

First multiply ① by 9 and ② by 4 to eliminate all of the fractions.
$9x^2 - 48x + 3y + 3 = 0$ ③
$-5x^2 - 6x + y - 2 = 0$ ④
I will use the substitution method because from ④ $y = 5x^2 + 6x + 2$. Substitute in ③.
$9x^2 - 48x + 3(5x^2 + 6x + 2) + 3 = 0$
$9x^2 - 48x + 15x^2 + 18x + 6 + 3 = 0$
$24x^2 - 30x + 9 = 0$
$(4x - 3)(6x - 3) = 0$
$x = 0.75$ or $x = 0.5$

Substitute in ② to determine y.

When $x = 0.75$:

$$-\frac{5}{4}(0.75)^2 - \frac{3}{2}(0.75) + \frac{1}{4}y - \frac{1}{2} = 0$$
$$\frac{1}{4}y = 2.328\,125$$
$$y = 9.3125$$

When $x = 0.5$:

$$-\frac{5}{4}(0.5)^2 - \frac{3}{2}(0.5) + \frac{1}{4}y - \frac{1}{2} = 0$$
$$\frac{1}{4}y = 1.5625$$
$$y = 6.25$$

The solutions are (0.75, 9.3125) and (0.5, 6.25).

Section 8.2 Page 452 Question 6

a) Alex's reasoning:

$n - m^2 = 7$ ① × 2 → $2n - 2m^2 = 14$ ③

$2m^2 - 2n = -1$ ② $\underline{2m^2 - 2n = -1}$ ②

Add ③ + ②: $0 \quad = 13$ which is impossible

Kaela's reasoning:

From ①, $n = m^2 + 7$. Substitute in ②.

$2m^2 - 2(m^2 + 7) = -1$

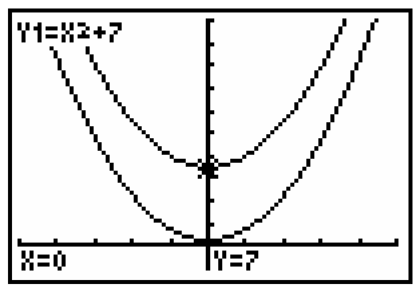
$-14 = -1$ which is impossible

They are both correct.

b) Write the equation using x and y instead of n and m and solve each for y.

Graph $y = x^2 + 7$ and $y = x^2 + 0.5$.

The two graphs have no point of intersection, so this confirms that the system has no solutions.



Section 8.2 Page 452 Question 7

a) Yes, Marie-Soleil's method works because subtracting is equivalent to multiplying the second equation by –1 and adding. The result will be the same.

b) *First System:*

$5x + 2y = 12$ ①

$x^2 - 2x + 2y = 7$ ②

Multiply ② by –1.

$-x^2 + 2x - 2y = -7$ ③

Add ① and ③.

$-x^2 + 7x = 5$

Second System:

$12m^2 - 4m - 8n = -3$ ①

$9m^2 - m - 8n = 2$ ②

Multiply ② by –1.

$-9m^2 + m + 8n = -2$ ③

Add ① and ③.

$3m^2 - 3m = -5$

These are the same equations as Marie-Soleil got.

c) Either is OK for me, but adding may be less error prone. With subtraction you have to be very careful with the signs of terms.

Section 8.2 Page 452 Question 8

$mx^2 - y = 16$ ①
$mx^2 + 2y = n$ ②

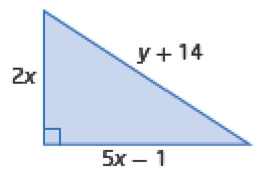
Given that (2, 8) is a solution, substitute $x = 2$ and $y = 8$.

In ①:
$m(2)^2 - 8 = 16$
$4m = 24$
$m = 6$

Then, in ②:
$6(2)^2 + 2(8) = n$
$n = 40$

Section 8.2 Page 452 Question 9

a) Perimeter $= 2x + 5x - 1 + y + 14$
$= 7x + y + 13$



b) Area $= = \frac{1}{2}$(base)(height)

$= \frac{1}{\cancel{2}}(5x - 1)(\cancel{2}^{1} x)$

$= 5x^2 - x$

c) Use the given facts, perimeter is 60 m and area is 10y square metres to write two equations.
$7x + y + 13 = 60$
$5x^2 - x = 10y$
Since the perimeter and the area are both based on the same dimensions, x and y must represent the same values. You can solve the system to find the actual dimensions.

d) From the first equation, $y = 47 - 7x$. Substitute in the second equation.
$5x^2 - x = 10(47 - 7x)$
$5x^2 + 69x - 470 = 0$
$(5x + 94)(x - 5) = 0$
Because x represents a length, reject the negative solution, so $x = 5$.
Substitute to determine y.
$y = 47 - 7(5)$
$y = 12$
Then, the base of the triangle is 24 m and its height is 10 m.

e) Using the results from part d), the hypotenuse is 26 cm. So, the perimeter is 10 + 24 + 26 or 60 m. This perimeter checks with the given information. The area is $\frac{1}{2}(24)(10)$ or 120 m^2. Also, $10y = 120$, so the area checks too.

Section 8.2 Page 453 Question 10

a) Let x represent the smaller integer and y the larger integer.

$x - y = -30$ ①

$y + 3 + x^2 = 189$ ②

b) From ①, $y = x + 30$. Substitute into ②.

$x + 30 + 3 + x^2 = 189$

$x^2 + x - 156 = 0$

$(x + 13)(x - 12) = 0$

$x = -13$ or $x = 12$

The two integers are 12 and 42 or −13 and 17.

c) Verify 12 and 42:

$x - y = 12 - 42$

$= -30$

$y + 3 + x^2 = 42 + 3 + 12^2$

$= 189$

Verify −13 and 17:

$x - y = -13 - 17$

$= -30$

$y + 3 + x^2 = 17 + 3 + (-13)^2$

$= 189$

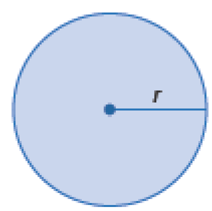
Both answers check.

Section 8.2 Page 453 Question 11

a) Let C represent the circumference.

Then, $C = 2\pi r$

Area: $C = 3\pi r^2$



b) Substitute from the first equation into the second.

$2\pi r = 3\pi r^2$

$2\pi r - 3\pi r^2 = 0$

$\pi r(2 - 3r) = 0$

Since the radius is a length, $r = \frac{2}{3}$.

Then, the circumference $C = 2\pi\left(\frac{2}{3}\right)$ or $\frac{4\pi}{3}$ centimetres and the area is

$\pi\left(\frac{2}{3}\right)^2$ or $\frac{4\pi}{9}$ square centimetres.

Section 8.2 Page 453 Question 12

a) kinetic energy: $E_k = \frac{5}{32}(d-20)^2$

potential energy: $E_p = -\frac{5}{32}(d-20)^2 + 62.5$

To determine the distance, when the kinetic energy is the same as the potential energy set $E_k = Ep$, solve:

$$\frac{5}{32}(d-20)^2 = -\frac{5}{32}(d-20)^2 + 62.5$$

$$\frac{\overset{5}{\cancel{10}}}{\underset{16}{\cancel{32}}}(d-20)^2 = 62.5$$

$$\begin{aligned}(d-20)^2 &= \frac{16}{5}(62.5)\\ (d-20)^2 &= 200\\ d-20 &= \pm\sqrt{200}\\ d &= 20 \pm 10\sqrt{2}\end{aligned}$$

The ball has the same amount of kinetic energy and potential energy at approximately 34.14 m and 5.86 m.

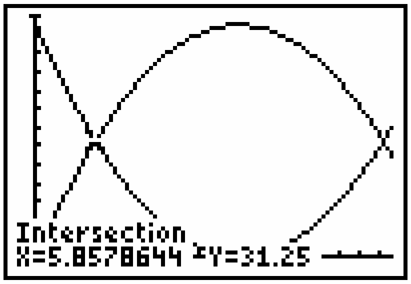
b) At $d = 20 \pm 10\sqrt{2}$,

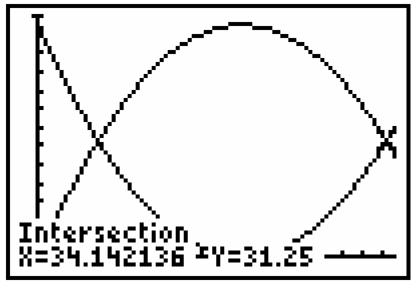
kinetic energy:
$$\begin{aligned} &= \frac{5}{32}(d-20)^2\\ &= \frac{5}{32}(20 \pm 10\sqrt{2} - 20)^2\\ &= \frac{5}{32}(\pm 10\sqrt{2})^2\\ &= \frac{5}{32}(200)\\ &= 31.25\end{aligned}$$

potential energy:
$$\begin{aligned} &= -\frac{5}{32}(d-20)^2 + 62.5\\ &= -\frac{5}{32}(20 \pm 10\sqrt{2} - 20)^2 + 62.5\\ &= -\frac{5}{32}(\pm 10\sqrt{2})^2 + 62.5\\ &= -\frac{5}{32}(200) + 62.5\\ &= 31.25\end{aligned}$$

At 5.86 m and 30.14 m both types of energy are 31.25 J.

c) Graph $y = \frac{5}{32}(x-20)^2$ and $y = -\frac{5}{32}(x-20)^2 + 62.5$.





The graph confirms that, to the nearest hundredth, the two types of energy are both 31.25 J at 5.86m and at 34.14 m.

d) Find the sum of the values of E_k and E_p at several choices for d. Observe that the sum is constant, 62.5. This can be deduced from the graph because each is a reflection of the other in the horizontal line $y = 31.25$.

Section 8.2 Page 453 Question 13

a) free-fall: $h(t) = -4.9t^2 + 2015$
with parachute: $h(t) = -10.5t + 980$
Solve the system of equations to determine at what value of t they intersect.
$-4.9t^2 + 2015 = -10.5t + 980$
$-4.9t^2 + 10.5t + 1035 = 0$
Use the quadratic formula with $a = -4.9$, $b = 10.5$, $c = 1035$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

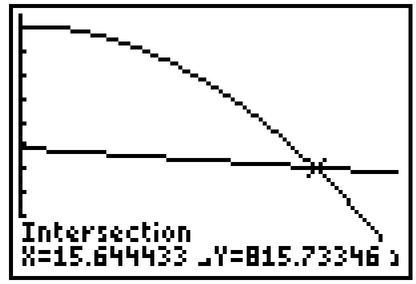
$$t = \frac{-10.5 \pm \sqrt{(10.5)^2 - 4(-4.9)(1035)}}{2(-4.9)}$$

$$t = \frac{-10.5 \pm \sqrt{20\ 396.25}}{-9.8}$$

$t = 15.644...$
The stuntman opens his parachute after 15.64 s, to the nearest hundredth of a second.

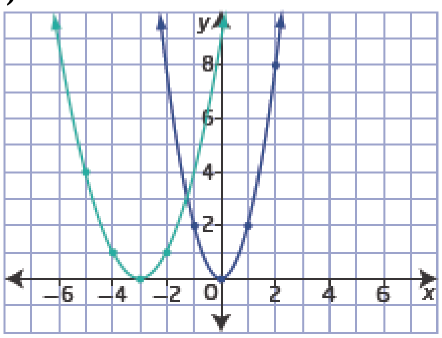
b) When $t = 15.644\ldots$,
$h(15.644\ldots) = -10.5(15.644\ldots) + 980$
$= 815.733\ldots$
The stuntman was about 815.73 m above the ground when he opened the parachute.

c) The result can be verified by sustituting into the two original equations.
Alternatively, using a graphing calculator, graph the two functions and find their point of intersection. The graph confirms the solutions found in parts a) and b).



Section 8.2 Page 454 Question 14

a)



b) The two parabolas intersect at about (–1.3, 3).

c) For the first quadratic, the vertex is at (0, 0). The equation has the form $y = ax^2$.
Substitute (1, 2) to determine the value of a.
$2 = a(1)^2$
$a = 2$
The equation for the first quadratic is $y = 2x^2$.

For the second quadratic, the vertex is at (–3, 0). The equation has the form $y = a(x + 3)^2$.
Substitute (–2, 1) to determine the value of a.
$1 = a(-2 + 3)^2$
$1 = a$
The equation for the second quadratic is $y = (x + 3)^2$.

A quadratic-quadratic system that models to two functions is $y = 2x^2$ and $y = (x + 3)^2$.

d) $y = 2x^2$ ①
$y = (x + 3)^2$ ②
Substitute from ① into ②.
$2x^2 = (x + 3)^2$
$2x^2 = x^2 + 6x + 9$
$x^2 - 6x - 9 = 0$
Use the quadratic equation with $a = 1$, $b = -6$, and $c = -9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-9)}}{2(1)}$$
$$x = \frac{6 \pm \sqrt{72}}{2}$$
$$x = \frac{6 \pm 6\sqrt{2}}{2}$$
$$x = 3 \pm 3\sqrt{2}$$
$x \approx 7.24$ and $x \approx -1.24$

Substitute in ① to determine y.

When $x = 3 + 3\sqrt{2}$:
$y = 2(3 + 3\sqrt{2})^2$
$y = 2(9 + 18\sqrt{2} + 18)$
$y = 54 + 36\sqrt{2}$
$y \approx 104.91$

When $x = 3 - 3\sqrt{2}$:
$y = 2(3 - 3\sqrt{2})^2$
$y = 2(9 - 18\sqrt{2} + 18)$
$y = 54 - 36\sqrt{2}$
$y \approx 3.09$

The solutions are approximately (7.24, 104.91) and (–1.24, 3.09).

The graph did not extend far enough to identify the point of intersection (7.24, 104.91). For the other point, (–1.24, 3.09), the estimate from the graph was reasonably close.

Section 8.2 Page 454 Question 15

a) For the first fragment, in $h(x) = -\frac{4.9}{(v_0 \cos\theta)^2}x^2 + (\tan\theta)x + h_0$ substitute $h_0 = 2500$, $\theta = 45°$, $v_0 = 60$.
$$h(x) = -\frac{4.9}{(60\cos 45°)^2}x^2 + (\tan 45°)x + 2500$$
$$h(x) \approx -0.003x^2 + x + 2500$$

For the second fragment, in $h(x) = -\frac{4.9}{(v_0 \cos\theta)^2}x^2 + (\tan\theta)x + h_0$ substitute $h_0 = 2500$, $\theta = 60°$, $v_0 = 60$.
$$h(x) = -\frac{4.9}{(60\cos 60°)^2}x^2 + (\tan 60°)x + 2500$$
$$h(x) \approx -0.005x^2 + 1.732x + 2500$$

b) $h(x) = -0.003x^2 + x + 2500$ ①
$h(x) = -0.005x^2 + 1.73x + 2500$ ②
Substitute from ① into ②.

$-0.003x^2 + x + 2500 = -0.005x^2 + 1.73x + 2500$

$0.002x^2 - 0.73x = 0$

$x(0.002x - 0.73) = 0$

$x = 0$ or $x = 365$

Substitute in ① to determine $h(x)$.

When $x = 0$:

$h(x) = 2500$

When $x = 365$:

$h(x) = -0.003(365)^2 + 365 + 2500$

$h(x) \approx 2465.33$

The solutions tell when the fragments were at the same height and distance from the volcano. The solution (0, 2500) is the position from which both fragments were blasted. The solution (365, 2465.33) means that each fragment was at a height of 365 m at a horizontal distance of 2465.33 m from the volcano.

Section 8.2 Page 454 Question 16

a) $h = -\frac{5}{1600}x^2 + 100$ is the parabolic path of the explosive, and $h = 1.19x$ is the linear profile of the mountainside. The solution for the system of equations will tell the horizontal distance from and the height above the base of the mountain, where the charge lands.

b) $h = -\frac{5}{1600}x^2 + 200$ ①

$h = 1.19x$ ②

Substitute from ① into ②.

$$-\frac{5}{1600}x^2 + 200 = 1.19x$$

$5x^2 + 1904x - 320\ 000 = 0$

Use the quadratic formula with $a = 5$, $b = 1904$, and $c = -320\ 000$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1904 \pm \sqrt{(1904)^2 - 4(5)(-320\ 000)}}{2(5)}$$

$$x = \frac{-1904 \pm \sqrt{10\ 025\ 216}}{10}$$

$x \approx 126.23$

Substitute in ② to determine h.

$h = 1.19(126.26215\ldots)$

$h \approx 150.21$

The explosive charge lands 150.21 m up the mountain, to the nearest hundredth of a metre.

Section 8.2 Page 455 Question 17

a) marginal revenue: $R_M = 5000 - 20x$

marginal cost: $C_M = 300 + \frac{1}{4}x^2$

For the maximum profit, set $R_M = C_M$ and solve.

$$5000 - 20x = 300 + \frac{1}{4}x^2$$

$$20\ 000 - 80x = 1200 + x^2$$

$$x^2 + 80x - 18\ 800 = 0$$

Use the quadratic formula with $a = 1$, $b = 80$, and $c = –18\ 800$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-80 \pm \sqrt{(80)^2 - 4(1)(-18\ 800)}}{2(1)}$$

$$x = \frac{-80 \pm \sqrt{81\ 600}}{2}$$

$$x = -40 \pm 10\sqrt{204}$$

$x \approx 103$, to the nearest whole number.

To maximize profit, 103 items should be sold.

b) Profit = total revenue – total cost

$$= 5000x - 10x^2 - \left(300x + \frac{1}{12}x^2\right)$$

Determine the profit when $x = 103$.

$$\text{Profit} = 5000(103) - 10(103)^2 - 300(103) - \frac{1}{12}(103)^2$$

$$= 377\ 125.92$$

The firm’s maximum monthly profit is \$377 125.92.

Section 8.2 Page 455 Question 18

a) $y = x^2 + 8x + 16$ ①

$y = x^2 - 8x + 16$ ②

$y = -\frac{x^2}{8} + 2$ ③

First find the point of intersection of ① and ②.

$x^2 + 8x + 16 = x^2 - 8x + 16$

$16x = 0$

$x = 0$

Substitute $x = 0$ in ① to determine y.

$y = 16$

The point of intersection of equations ① and ② is (0, 16).

Next, find the point of intersection of ① and ③.

$$x^2 + 8x + 16 = -\frac{x^2}{8} + 2$$
$$8x^2 + 64x + 128 = -x^2 + 16$$
$$9x^2 + 64x + 112 = 0$$

Use the quadratic formula with $a = 9$, $b = 64$, and $c = 112$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-64 \pm \sqrt{(64)^2 - 4(9)(112)}}{2(9)}$$
$$x = \frac{-64 \pm \sqrt{64}}{18}$$
$$x = \frac{-64 \pm 8}{18}$$

$x = –4$ or $x \approx –3.11$

Substitute in ① to determine y.

When $x = –4$:
$y = (–4)^2 + 8(–4) + 16$
$y = 0$

When $x = 3.11$:
$y = (–3.11\ldots)^2 + 8(–3.11\ldots) + 16$
$y \approx 0.79$

The points of intersection of ① and ③ are (–4, 0) and (–3.11, 0.79).

Next, find the point of intersection of ② and ③.

$$x^2 - 8x + 16 = -\frac{x^2}{8} + 2$$
$$8x^2 - 64x + 128 = -x^2 + 16$$
$$9x^2 - 64x + 112 = 0$$

$(9x – 28)(x – 4) = 0$

$x \approx 3.11$ or $x = 4$

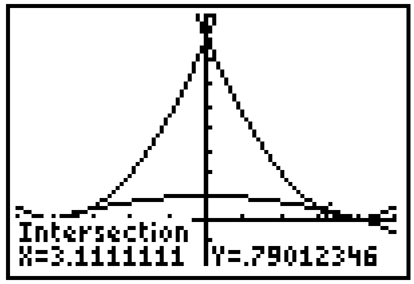
Substitute in ② to determine y.

When $x = 3.11$:
$y = (3.11\ldots)^2 – 8(3.11\ldots) + 16$
$y \approx 0.79$

When $x = 4$:
$y = 4^2 – 8(4) + 16$
$y = 0$

The points of intersection of ② and ③ are (3.11, 0.79) and (4, 0).

Use a graph to determine which points form the triangular sail.
The coordinates of the vertices of the sail are (–3.11, 0.79), (0, 16) and (3.11, 0.79).



b) Example:
The base of the sail is approximately 6 m and its height is approximately 15 m.

$$\begin{aligned}\text{Area of triangle } &= \frac{1}{2}\times\text{base}\times\text{height}\\ &= \frac{1}{2}(6)(15)\\ &\approx 45\end{aligned}$$

An estimate of the area of material required to make the sail is 50 m^2, rounded up because each length was rounded down and allowing a bit extra for seams.

Section 8.2 Page 455 Question 19

a) Determine the point of intersection of the tangent line $y = 4x - 2$ and the parabola $y = 2x^2 - 4x + 6$.

$$\begin{aligned}4x - 2 &= 2x^2 - 4x + 6\\ 0 &= 2x^2 - 8x + 8\\ 0 &= x^2 - 4x + 4\\ 0 &= (x - 2)(x - 2)\\ x &= 2\end{aligned}$$

Substitute to determine y.
$y = 4(2) - 2$
$y = 6$
The coordinates of A are (2, 6).

b) The slope of the tangent line $y = 4x - 2$ is 4. The slope of any line perpendicular to the tangent line is $-\frac{1}{4}$. So, the equation of the normal line will have the form $y = -\frac{1}{4}x + b$.
Use the coordinates of A to determine the value of b.
Substitute (2, 6).

$$6 = -\frac{1}{4}(2) + b$$

$$b = \frac{13}{2}$$

The equation of the normal line at the point A is $y = -\frac{1}{4}x + \frac{13}{2}$.

c) Determine the coordinates of the points of intersection of $y = -\frac{1}{4}x + \frac{13}{2}$ and $y = 2x^2 - 4x + 6$.

$$\begin{aligned}
-\frac{1}{4}x + \frac{13}{2} &= 2x^2 - 4x + 6 \\
-x + 26 &= 8x^2 - 16x + 24 \\
0 &= 8x^2 - 15x - 2 \\
0 &= (8x + 1)(x - 2) \\
x &= -0.125 \text{ or } x = 2
\end{aligned}$$

Substitute $x = -0.125$ to determine the y-coordinate of B.

$$y = -\frac{1}{4}(-0.125) + \frac{13}{2}$$

$$y = 6.531\ 25$$

The coordinates of B are (–0.125, 6.531 25).

Use the Pythagorean formula to find the distance AB.

$AB^2 = [2 - (-0.125)]^2 + [6 - 6.531\ 25]^2$

$AB \approx 2.19$

The length of the chord AB is 2.19 units, to the nearest hundredth.

Section 8.2 Page 455 Question 20

$$y = \frac{2x - 1}{x}$$

$$\frac{x}{x + 2} + y - 2 = 0$$

Substitute from the first equation into the second.

$$\begin{aligned}
\frac{x}{x + 2} + \frac{2x - 1}{x} - 2 &= 0 \\
x^2 + (x + 2)(2x - 1) - 2x(x + 2) &= 0 \\
x^2 + 2x^2 + 3x - 2 - 2x^2 - 4x &= 0 \\
x^2 - x - 2 &= 0 \\
(x - 2)(x + 1) &= 0
\end{aligned}$$

Therefore, $x = 2$ or $x = -1$.

Substitute into the first equation to find the corresponding y-values.

When $x = 2$:

$$y = \frac{2(2) - 1}{2}$$

$$y = 1.5$$

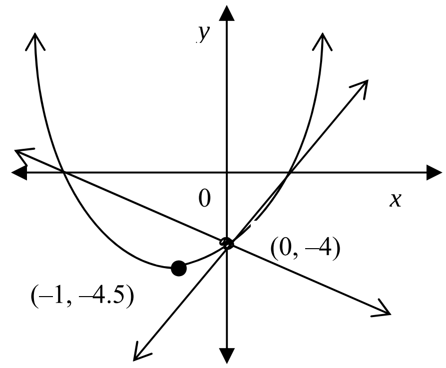
When $x = -1$:

$$y = \frac{2(-1) - 1}{-1}$$

$$y = 3$$

The solution to the system is (2, 1.5) and (–1, 3).

Section 8.2 Page 455 Question 21



Draw a diagram to show the given information. The vertex of the parabola is at (–1, –4.5), so its equation has the form $y = a(x + 1)^2 – 4.5$. Use the point (0, –4) to determine the value of a.

$-4 = a(0 + 1)^2 - 4.5$
$a = 0.5$

The equation for the parabola is $y = 0.5(x + 1)^2 – 4.5$.

Next, determine where the parabola intersects the x-axis. Solve:

$0 = 0.5(x + 1)^2 - 4.5$
$9 = (x + 1)^2$
$\pm 3 = x + 1$
$x = 2$ or $x = -4$

Now, determine the equation of each line.

Through (0, –4) and (2, 0):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-4 - 0}{0 - 2}$$
$$m = 2$$

The equation is $y = 2x – 4$.

Through (0, –4) and (–4, 0):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-4 - 0}{0 - (-4)}$$
$$m = -1$$

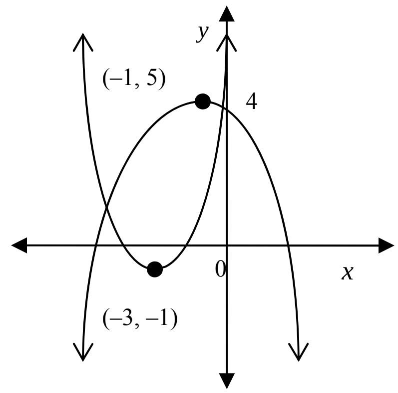
The equation is $y = –x – 4$.

The linear-quadratic system is $y = 0.5(x + 1)^2 – 4.5$ and $y = 2x – 4$ or $y = 0.5(x + 1)^2 – 4.5$ and $y = –x – 4$.

Section 8.2 Page 455 Question 22

Example: Graphing is relatively quick using a graphing calculator, may be time-consuming and inaccurate using pencil and grid paper. Sometimes, rearranging the equation to enter into the calculator is not easy. The algebraic methods will always give an exact answer and do not rely on having technology available. Some systems of equations may be faster to solve algebraically, especially if one variable is easily eliminated.

Section 8.2 Page 455 Question 23



Draw a sketch to help visualize the given information.
Determine the equation of the parabola with vertex at (–3, –1) and passing through (–2, 1).
The equation has the form $y = a(x + 3)^2 - 1$.
Substitute (–2, 1) to determine the value of a.
$1 = a(-2 + 3)^2 - 1$
$2 = a$
The equation for this parabola is
$y = 2(x + 3)^2 - 1$.

Next, determine the equation of the parabola with vertex at (–1, 5) and y-intercept 4.
The equation has the form $y = a(x + 1)^2 + 5$.
Substitute (0, 4) to determine the value of a.
$4 = a(0 + 1)^2 + 5$
$-1 = a$
The equation for this parabola is $y = -(x + 1)^2 + 5$.

Solve the quadratic-quadratic system to determine the coordinates of the points of intersection of the two parabolas.

$$2(x + 3)^2 - 1 = -(x + 1)^2 + 5$$
$$2(x^2 + 6x + 9) - 1 = -(x^2 + 2x + 1) + 5$$
$$3x^2 + 14x + 13 = 0$$

Use the quadratic formula with $a = 3$, $b = 14$ and $c = 13$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-14 \pm \sqrt{(14)^2 - 4(3)(13)}}{2(3)}$$
$$x = \frac{-14 \pm \sqrt{40}}{6} \text{ or } \frac{-7 \pm \sqrt{10}}{3}$$

$x \approx -1.28$ or $x \approx -3.38$
Substitute to find the corresponding y-values.

When $x = \dfrac{-7 + \sqrt{10}}{3}$:

When $x = \dfrac{-7 - \sqrt{10}}{3}$:

$$y = 2\left(\frac{-7+\sqrt{10}}{3}+3\right)^2 - 1$$
$$y = 2\left(\frac{2+\sqrt{10}}{3}\right)^2 - 1$$
$$y = 2\left(\frac{4+4\sqrt{10}+10}{9}\right) - 1$$
$$y = \frac{19+8\sqrt{10}}{9}$$
$y \approx 4.92$

$$y = 2\left(\frac{-7-\sqrt{10}}{3}+3\right)^2 - 1$$
$$y = 2\left(\frac{2-\sqrt{10}}{3}\right)^2 - 1$$
$$y = 2\left(\frac{4-4\sqrt{10}+10}{9}\right) - 1$$
$$y = \frac{19-8\sqrt{10}}{9}$$
$y \approx -0.70$

The coordinates of the points of intersection of the two parabolas are approximately (–1.28, 4.92) and (–3.39, –0.70).

Section 8.2 Page 456 Question 24

Example: Substitute $y = -\frac{1}{2}x - 2$ into $y = x^2 - 4x + 2$.

$$-\frac{1}{2}x - 2 = x^2 - 4x + 2$$
$$0 = 2x^2 - 7x + 8$$

Use the quadratic formula with $a = 2$, $b = -7$, and $c = 8$.

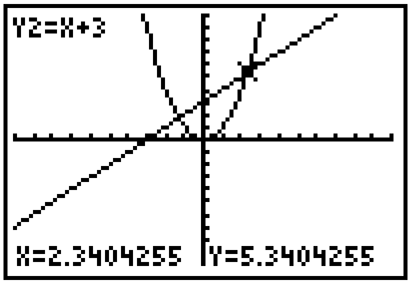
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(8)}}{2(2)}$$
$$x = \frac{7 \pm \sqrt{-15}}{4}$$

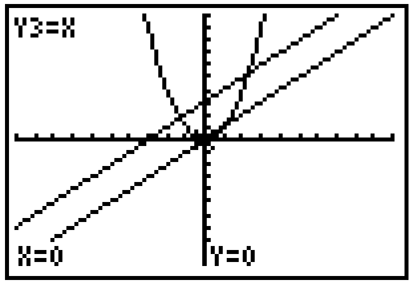
The equation has no real roots.

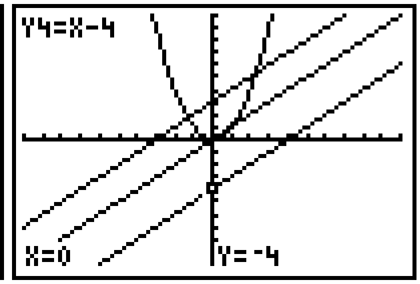
Therefore, the graphs of $y = -\frac{1}{2}x - 2$ and $y = x^2 - 4x + 2$ do not intersect.

Section 8.2 Page 456 Question 25

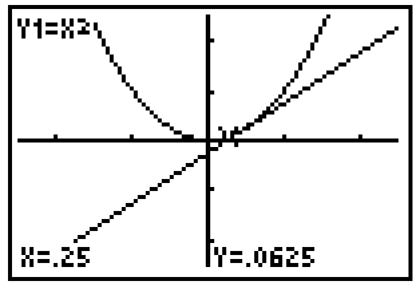
Step 1 From experimental graphs it appears that there are two solutions when $b \geq 0$, one solution when $b = 0.25$, and no solution when $b < 0.25$.







When you zoom in on the second screen, you can see that there are actually two solutions when $b = 0$. When $b = –0.25$ there is only one solution.



Step 2 Solve the linear-quadratic system $y = x^2$ and $y = x + b$.

$x^2 = x + b$

$x^2 - x - b = 0$

Use the quadratic formula with $a = 1$, $b = –1$, and $c = –b$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-b)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 4b}}{2}$$

The expression under the radical determines when there are two, one, or no solutions.

When $1 + 4b > 0$, there are two solutions. This is when $b > -\frac{1}{4}$.

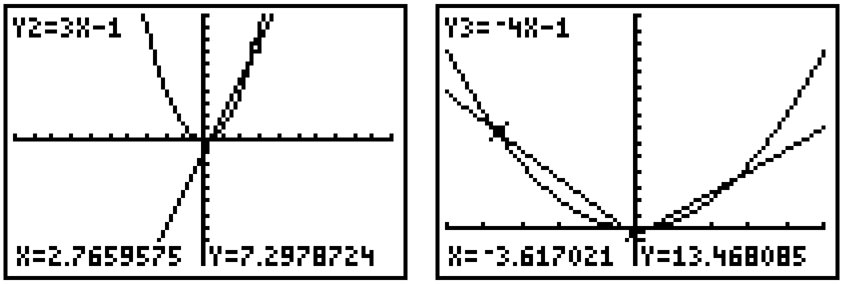
When $1 + 4b = 0$, there is only one solution. This is when $b = -\frac{1}{4}$.

When $1 + 4b < 0$ there are no real solutions. This is when $b < -\frac{1}{4}$.

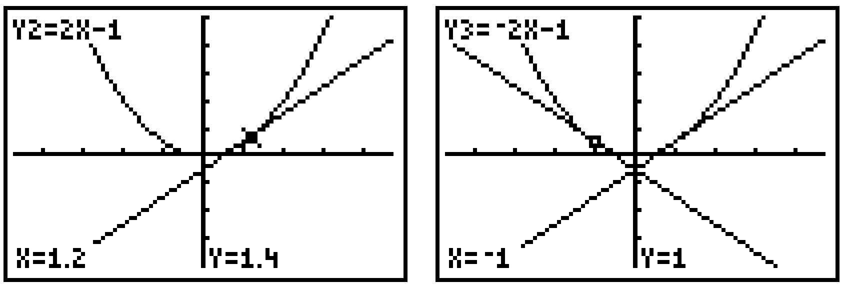
So, the experimental conclusion in Step 1 was not accurate. In fact, there are two solutions when $b > -\frac{1}{4}$, one solution when $b = -\frac{1}{4}$, and no solution when $b < -\frac{1}{4}$.

Step 3 It seems that there are two solutions when $|m| > 2$, one solution when $m = \pm 2$, and no solution when $|m| < 2$.

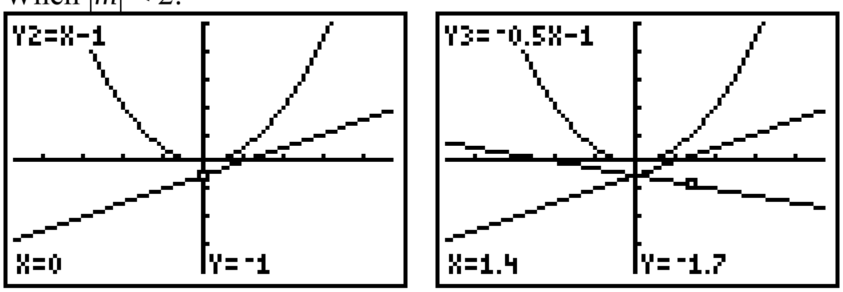
When $|m| > 2$:



When $m = \pm 2$:



When $|m| < 2$:



Step 4 Solve the linear-quadratic system $y = x^2$ and $y = mx - 1$.
$x^2 = mx - 1$
$x^2 - mx + 1 = 0$
Use the quadratic formula with $a = 1$, $b = -m$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)}}{2(1)}$$

$$x = \frac{m \pm \sqrt{m^2 - 4}}{2}$$

The expression under the radical determines when there are two, one, or no solutions.
When $m^2 - 4 > 0$ there are two solutions. This is true when $m^2 > 4$ or $|m| > 2$.
When $m^2 - 4 = 0$ there is one solution. This is true when $m^2 = 4$ or $m = \pm 2$.
When $m^2 - 4 < 0$ there are no solutions. This is true when $m^2 < 4$ or $|m| < 2$.
The prediction from step 2 is correct: there are two solutions when $|m| > 2$, one solution when $m = \pm 2$, and no solution when $|m| < 2$.

Step 5 Solve the linear-quadratic system $y = x^2$ and $y = mx + b$.
$x^2 = mx + b$

$x^2 - mx - b = 0$

Use the quadratic formula with $a = 1$, $b = -m$, and $c = -b$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-m) \pm \sqrt{(-m)^2 - 4(1)(-b)}}{2(1)}$$

$$x = \frac{m \pm \sqrt{m^2 + 4b}}{2}$$

The expression under the radical determines when there are two, one, or no solutions. The case when $m = 1$ was examined in Steps 1 and 2. The case when $b = -1$ was examined in Steps 3 and 4.

When $m^2 + 4b > 0$ there are two solutions. This is true when $m^2 > -4b$, *i.e.*, for $b < 0$ and $|m| > 2\sqrt{-b}$ or for $b \geq 0$ and any real value for m, $m \neq 0$ if $b = 0$

When $m^2 + 4b = 0$, there is one solution. This is true when $m^2 = -4b$, *i.e.*, for $b < 0$ and $|m| = 2\sqrt{-b}$.

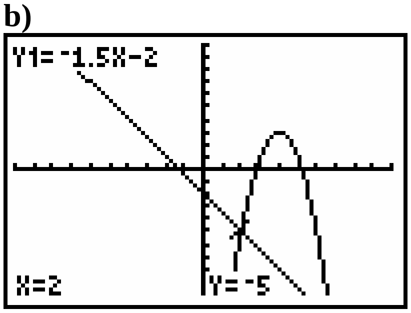
When $m^2 + 4b < 0$, there is no solution. This is true when $m^2 < -4b$ or for $b < 0$ and $|m| < 2\sqrt{-b}$.

Chapter 8 Review

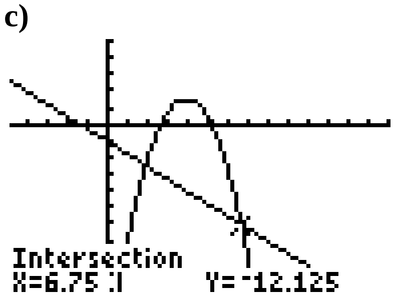
Chapter 8 Review Page 457 Question 1

a) Both tables of values include the point (2, –5), so this is a solution to the system of equations $y = -1.5x - 2$ and $y = -2(x - 4)^2 + 3$.

b)



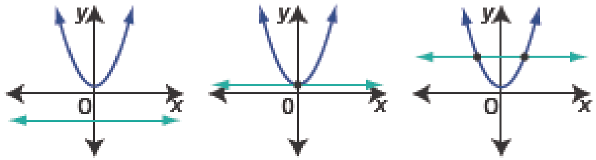
c)



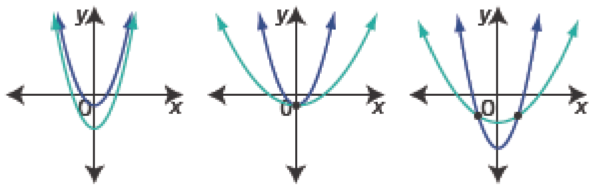
The other solution is (6.75, –12.125).

Chapter 8 Review Page 457 Question 2

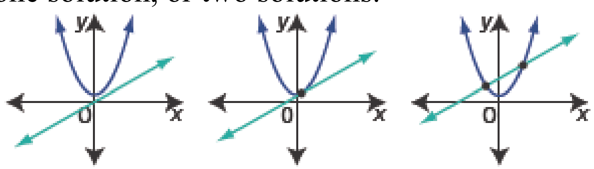
a) A system involving a parabola and a horizontal line can have no solution, one solution, or two solutions.



b) A system involving two parabolas that both open upward can have no solution, one solution, or two solutions.



c) A system involving a parabola and a line with a positive slope can have no solution, one solution, or two solutions.

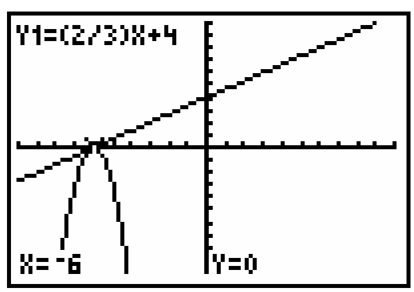


Chapter 8 Review Page 457 Question 3

a) $y = \frac{2}{3}x + 4$

$y = -3(x + 6)^2$

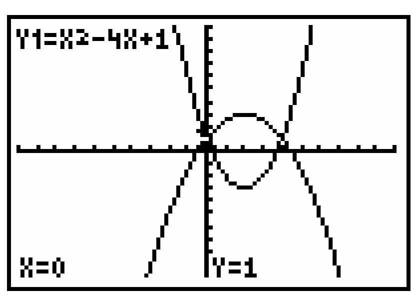
The solution is (–6, 0).

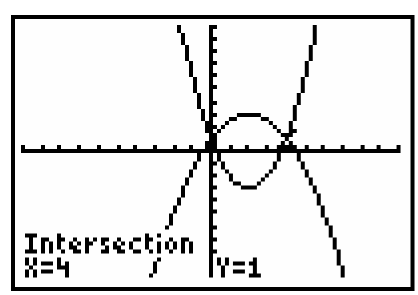


b) $y = x^2 - 4x + 1$

$y = -\frac{1}{2}(x - 2)^2 + 3$

The solutions are (0, 1) and (4, 1).





Chapter 8 Review Page 457 Question 4

Example: No, Adam is not correct. The vertex of $y = x^2 + 1$ is at (0, 1) and the vertex of $y = x^2 + 3$ is at (0, 3). The second parabola is a vertical translation 3 units upward of the first; they will never intersect.

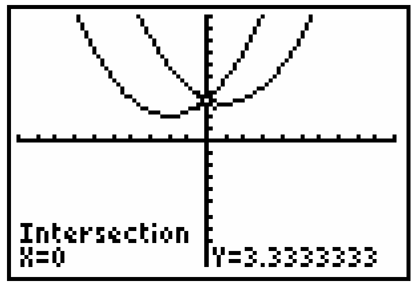
Chapter 8 Review Page 457 Question 5

a) $p = \frac{1}{3}(x+2)^2 + 2$

$p = \frac{1}{3}(x-1)^2 + 3$

The solution is (0, 3.33) or

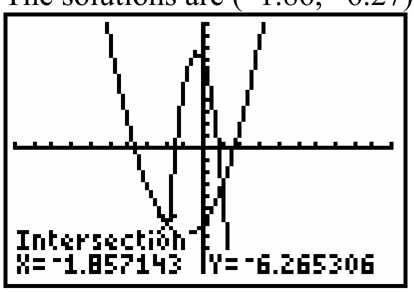
$x = 0$, $p = 3\frac{1}{3}$.

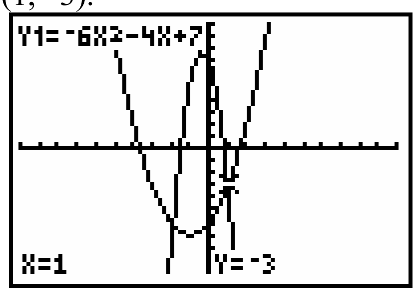


b) $y = -6x^2 - 4x + 7$

$y = x^2 + 2x - 6$

The solutions are (–1.86, –6.27) and (1, –3).



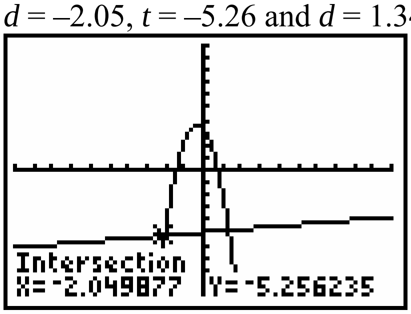


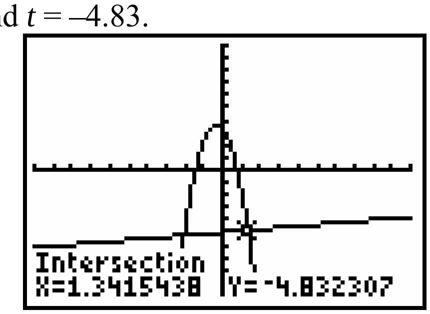
c) $t = -3d^2 - 2d + 3.25$

$t = \frac{1}{8}d - 5$

The solutions are (–2.05, –5.26) and (1.34, –4.83) or

$d = -2.05$, $t = -5.26$ and $d = 1.34$ and $t = -4.83$.





Chapter 8 Review Page 457 Question 6

a) For the road arch:
The vertex is at (8, 6) and (0, 0) is a point.
The equation, in vertex form, is $y = a(x - 8)^2 + 6$.
Substitute (0, 0) to determine the value of a.

$0 = a(0 - 8)^2 + 6$

$a = -\frac{3}{32}$

The equation for the road arch is $y = -\frac{3}{32}(x-8)^2 + 6$.

For the river arch:
The vertex is at (24, 8) and a point is (36, 0).
The equation, in vertex form, is $y = a(x - 24)^2 + 8$.
Substitute (36, 0) to determine the value of a.

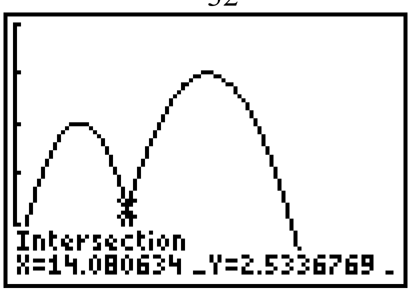
$0 = a(36 - 24)^2 + 8$

$-8 = 144a$

$a = -\frac{1}{18}$

The equation for the river arch is $y = -\frac{1}{18}(x-24)^2 + 8$.

b) Graph $y = -\frac{3}{32}(x-8)^2 + 6$ and $y = -\frac{1}{18}(x-24)^2 + 8$



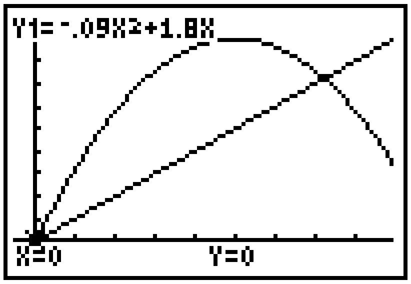
The solution is (14.08, 2.53).

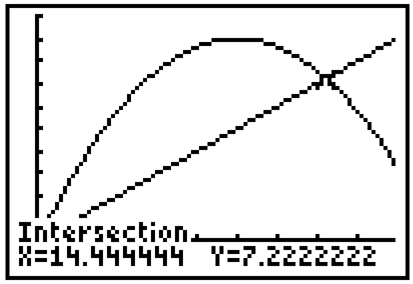
c) Example: The intersection tells the engineer that the distance of the footing from the other side of the road arch is14.08 m, and its height is 2.53 m.

Chapter 8 Review Page 458 Question 7

a) Example: The first equation models the horizontal distance travelled, d, and the height of the ball, h; it follows a parabolic path that opens downward. The linear equation models the profile of the hill with a constant slope, using d and h in the same way as the first equation.

b) Graph the two functions to find their point of intersection.
$h = -0.09d^2 + 1.8d$
$h = \frac{1}{2}d$





The solutions to the system are (0, 0) and (14.44, 7.22).

c) Example: The point (0, 0) represents the starting point, where the ball is kicked. The point (14.44, 7.22) is where the ball lands on the hill. The coordinates give the horizontal distance and vertical distance from the point that the ball is kicked.

Chapter 8 Review Page 458 Question 8

a) Example: (2, –3) and (6, 5).

b) $y = 2x - 7$ ①
$y = x^2 - 6x + 5$ ②
Substitute from ① into ②.
$2x - 7 = x^2 - 6x + 5$
$0 = x^2 - 8x + 12$
$0 = (x - 2)(x - 6)$
Then, $x = 2$ or $x = 6$.
Substitute into ① to determine the corresponding y-values.

When $x = 2$:	When $x = 6$:
$y = 2(2) - 7$	$y = 2(6) - 7$
$y = -3$	$y = 5$

The solutions to the system are (2, –3) and (6, 5).

Chapter 8 Review Page 458 Question 9

$4m^2 - 3n = -2$ ①
$m^2 + \frac{7}{2}m + 5n = 7$ ②
Verify $\left(\frac{1}{2}, 1\right)$:

Substitute $m = \frac{1}{2}$ and $n = 1$.

In ①:

$$\text{Left Side} = 4\left(\frac{1}{2}\right)^2 - 3(1)$$
$$= -2$$
$$= \text{Right Side}$$

In ②:

$$\text{Left Side} = m^2 + \frac{7}{2}m + 5n$$
$$= \left(\frac{1}{2}\right)^2 + \frac{7}{2}\left(\frac{1}{2}\right) + 5(1)$$
$$= \frac{1}{4} + \frac{7}{4} + 5$$
$$= 7$$
$$= \text{Right Side}$$

The solution $\left(\frac{1}{2},\ 1\right)$ is correct.

Chapter 8 Review Page 458 Question 10

a) $p = 3k + 1$ ①
$p = 6k^2 + 10k - 4$ ②

Substitute from ① into ②. This method is chosen because ① is already solved for p.

$3k + 1 = 6k^2 + 10k - 4$
$0 = 6k^2 + 7k - 5$
$0 = (3k + 5)(2k - 1)$

So, $k = -\frac{5}{3}$ or $k = \frac{1}{2}$.

Substitute into ① to determine the corresponding p-values.

When $k = -\frac{5}{3}$:

$$p = 3\left(-\frac{5}{3}\right) + 1$$
$$= -4$$

When $k = \frac{1}{2}$:

$$p = 3\left(\frac{1}{2}\right) + 1$$
$$= \frac{5}{2}$$

The solutions are $\left(-\frac{5}{3},\ -4\right)$ and $\left(\frac{1}{2},\ \frac{5}{2}\right)$.

b) $4x^2 + 3y = 1$ ①
$3x^2 + 2y = 4$ ②

Use the elimination method as it is relatively easy to make opposite coefficients for the y-terms.

Multiply ① by 2 and ② by –3.

$8x^2 + 6y = 2$ ③
$-9x^2 - 6y = -12$ ④

Add ③ and ④.

$-x^2 = -10$

$x = \pm\sqrt{10}$ or $x \approx \pm 3.16$

Substitute into ② to find the corresponding y-values.

When $x = \sqrt{10}$:

$$3(\sqrt{10})^2 + 2y = 4$$
$$2y = 4 - 30$$
$$y = -13$$

When $x = -\sqrt{10}$:

$$3(-\sqrt{10})^2 + 2y = 4$$
$$2y = 4 - 30$$
$$y = -13$$

The solutions are approximately (3.16, –13) and (–3.16, –13).

c) $\frac{w^2}{2} + \frac{w}{4} - \frac{z}{2} = 3$ ①

$\frac{w^2}{3} - \frac{3w}{4} + \frac{z}{6} + \frac{1}{3} = 0$ ②

First multiply each equation by its LCM to eliminate fractions.

Multiply ① by 4 and ② by 12.

$2w^2 + w - 2z = 12$ ③

$4w^2 - 9w + 2z + 4 = 0$ ④

Now it is clear that z is eliminated by adding ③ and ④.

$6w^2 - 8w + 4 = 12$

$6w^2 - 8w - 8 = 0$

$3w^2 - 4w - 4 = 0$

$(3w + 2)(w - 2) = 0$

$w = -\frac{2}{3}$ or $w = 2$

Substitute into ① to find the corresponding z-values.

When $w = -\frac{2}{3}$:

$$\frac{1}{2}\left(-\frac{2}{3}\right)^2 + \frac{1}{4}\left(-\frac{2}{3}\right) - \frac{z}{2} = 3$$
$$\frac{2}{9} - \frac{1}{6} - \frac{z}{2} = 3$$
$$\frac{4-3}{18} - 3 = \frac{z}{2}$$
$$-\frac{53}{18} = \frac{z}{2}$$
$$z = -\frac{53}{9}$$

When $w = 2$:

$$\frac{2^2}{2} + \frac{2}{4} - \frac{z}{2} = 3$$
$$2 + \frac{1}{2} - \frac{z}{2} = 3$$
$$\frac{5}{2} - 3 = \frac{z}{2}$$
$$-\frac{1}{2} = \frac{z}{2}$$
$$z = -1$$

The solutions are $\left(-\frac{2}{3}, -\frac{53}{9}\right)$ and (2, –1).

d) $2y - 1 = x^2 - x$ ①
$x^2 + 2x + y - 3 = 0$ ②
From equation ② y can be easily isolated, then substituted into ①.
$y = 3 - x^2 - 2x$
$2(3 - x^2 - 2x) - 1 = x^2 - x$
$6 - 2x^2 - 4x - 1 = x^2 - x$
$0 = 3x^2 + 3x - 5$
Use the quadratic formula with $a = 3$, $b = 3$, and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{-3 \pm \sqrt{69}}{6}$$

$x \approx 0.88$ or $x \approx -1.88$
Substitute in ① to determine the corresponding y-values.

When $x = \dfrac{-3 + \sqrt{69}}{6}$:

$$2y - 1 = \left(\frac{-3 + \sqrt{69}}{6}\right)^2 - \frac{-3 + \sqrt{69}}{6}$$

$$2y = 1 + \frac{9 - 6\sqrt{69} + 69 - 6(-3 + \sqrt{69})}{36}$$

$$2y = \frac{36 + 9 + 69 + 18 - 12\sqrt{69}}{36}$$

$$2y = \frac{132 - 12\sqrt{69}}{36}$$

$$y = \frac{11 - \sqrt{69}}{6} \approx 0.45$$

When $x = \dfrac{-3 - \sqrt{69}}{6}$:

$$2y - 1 = \left(\frac{-3 - \sqrt{69}}{6}\right)^2 - \frac{-3 - \sqrt{69}}{6}$$

$$2y = 1 + \frac{9 + 6\sqrt{69} + 69 - 6(-3 - \sqrt{69})}{36}$$

$$2y = \frac{36 + 9 + 69 + 18 + 12\sqrt{69}}{36}$$

$$2y = \frac{132 + 12\sqrt{69}}{36}$$

$$y = \frac{11 + \sqrt{69}}{6} \approx 3.22$$

The solutions are approximately (0.88, 0.45) and (–1.88, 3.22).

Chapter 8 Review Page 458 Question 11

a) Solve $h(d) = -0.002d^2 + 0.3d$ ①
and $h(d) = -0.004d^2 + 0.5d$ ②
Substitute from ① into ②.
$-0.002d^2 + 0.3d = -0.004d^2 + 0.5d$
$0.004d^2 - 0.002d^2 + 0.3d - 0.5d = 0$
$0.002d^2 - 0.2d = 0$
$d(0.002d - 0.2) = 0$
$d = 0$ or $d = 100$

The ball is at the same height when the distance is 0 m, that is before the ball is hit, and when it is 100 m.

b) When $d = 0$, $h = 0$; the height is 0 m before the ball is hit.
When $d = 100$:
$h = -0.002(100)^2 + 0.3(100)$
$h = 10$
When the distance is 100 m, the height of the ball is 10 m for either golf club.

Chapter 8 Review   Page 458   Question 12

a) Example: By solving the related system of equations, the scientists would determine the time when both cultures have the same rate of increase of surface area.

b) $S(t) = -0.007t^2 + 0.05t$ ①
$S(t) = -0.0085t^2 + 0.06t$ ②
Substitute for $S(t)$ from ① into ②.
$-0.007t^2 + 0.05t = -0.0085t^2 + 0.06t$
$0.0015t^2 - 0.01t = 0$
$t(0.0015t - 0.01) = 0$
$t = 0$ or $t \approx 6.67$

When $t = 0$:
$S(t) = 0$

When $t \approx 6.67$
$S(t) = -0.007(6.666\ldots)^2 + 0.05(6.666\ldots)$
$S(t) \approx 0.02$

The solutions to the system are (0, 0) and approximately (6.67, 0.02).

c) The solution $t = 0$, $S(t) = 0$ is the start of the experiment, before any growth has occurred. The other solution means that in 6.67 h, or 6h 40 min, the surface area of both cultures is increasing at the same rate, 0.02 mm^2/h.

Chapter 8 Practice Test   Page 459   Question 1

Visualize the graphs extended in quadrant I, they will intersect.
The best answer is **C**, the system has solutions in quadrant I and II only.

Chapter 8 Practice Test   Page 459   Question 2

Given that $y = \frac{1}{2}(x-6)^2 + 2$ and $y = 2x + k$ have no solution, then

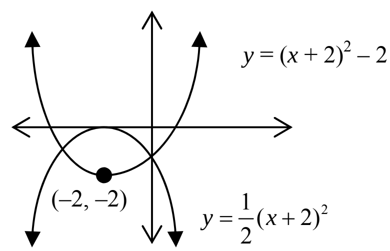
$y = -\frac{1}{2}(x-6)^2 + 2$ and $y = 2x + k$ must have two solutions because the parabola is the reflection in a horizontal line through the vertex of the original one. If the line did not intersect the first parabola at all it must intersect the reflected parabola twice.
C is the best answer.

Chapter 8 Practice Test Page 459 Question 3

The tables of values have two points in common (2, –3) and (4, –3), so the two quadratic functions must have at least these two real solutions.
Answer **B** is the best.

Chapter 8 Practice Test Page 459 Question 4

$y = (x + 2)^2 - 2$ ①
$y = \frac{1}{2}(x + 2)^2$ ②



A sketch shows that there must be two solutions. Use the substitution method to determine whether answer C or D is best.
Substitute from ① into ②.

$(x + 2)^2 - 2 = \frac{1}{2}(x + 2)^2$

$\frac{1}{2}(x + 2)^2 = 2$

$x^2 + 4x + 4 = 4$

$x^2 + 4x = 0$

$x(x + 4) = 0$

$x = 0$ or $x = -4$

Answer **D** is best.

Chapter 8 Practice Test Page 459 Question 5

Connor put incorrect signs in the brackets in line 4.
Line 4 should be $(2m - 5)(m + 3) = 0$.
Answer **D** is best.

Chapter 8 Practice Test Page 459 Question 6

$4x^2 - my^2 = 10$ ①
$mx^2 + ny^2 = 20$ ②
Substitute $x = 2$ and $y = 1$ into ①:
$4(2)^2 - m(1)^2 = 10$
$16 - 10 = m$
$m = 6$
Substitute $x = 2$, $y = 1$ and $m = 6$ into ②:
$6(2)^2 + n(1)^2 = 20$
$n = 20 - 24$
$n = -4$

Chapter 8 Practice Test Page 459 Question 7

a) $5x^2 + 3y = -3 - x$
$2x^2 - x = -4 - 2y$
First rearrange the terms.
$5x^2 + x + 3y = -3$ ①
$2x^2 - x + 2y = -4$ ②
Multiply ① by 2 and ② by –3.
$10x^2 + 2x + 6y = -6$ ③
$\underline{-6x^2 + 3x - 6y = 12}$ ④
$4x^2 + 5x \quad = 6$ ③ + ④
$4x^2 + 5x - 6 = 0$
$(4x - 3)(x + 2) = 0$
$x = \frac{3}{4}$ or $x = -2$
Substitute in ② to determine the corresponding y-values.

When $x = \frac{3}{4}$:

$$2\left(\frac{3}{4}\right)^2 - \frac{3}{4} + 4 = -2y$$
$$\frac{9}{8} - \frac{3}{4} + 4 = -2y$$
$$\frac{9 - 6 + 32}{8} = -2y$$
$$\frac{35}{8} = -2y$$
$$y = -\frac{35}{16}$$

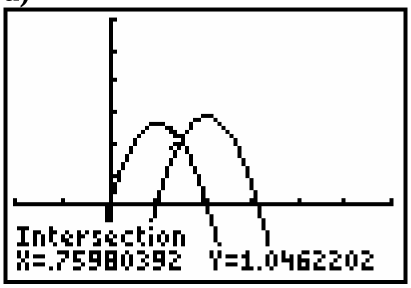
When $x = -2$:
$2(-2)^2 - (-2) + 4 = -2y$
$14 = -2y$
$y = -7$

The solutions are $\left(\frac{3}{4}, -\frac{35}{16}\right)$ and (–2, –7).

b) $y = 7x - 11$ ①
$5x^2 - 3x - y = 6$ ②
Substitute for y from ① into ②.
$5x^2 - 3x - (7x - 11) = 6$
$5x^2 - 3x - 7x + 11 - 6 = 0$
$5x^2 - 10x + 5 = 0$
$x^2 - 2x + 1 = 0$
$(x - 1)(x - 1) = 0$
Therefore, $x = 1$
Substitute into ① to determine the corresponding value of y.
$y = 7(1) - 11$
$y = -4$
The solution is (1, –4).

Chapter 8 Practice Test Page 460 Question 8

a)



The coordinates of the point of intersection are approximately (0.76, 1.05).

b) Example: 0.76 s after Sophie starts her jump, both dancers are at the same height, 1.05 m.

Chapter 8 Practice Test Page 460 Question 9

a) Perimeter: $2(x + 8 + x + 6) = 8y$
$4x + 28 = 8y$
Area: $(x + 8)(x + 6) = 6y + 3$
$x^2 + 14x + 48 = 6y + 3$

b) Solve the system:
$x + 7 = 2y$ ①
$x^2 + 14x + 45 = 6y$ ②
Substitute $2y = x + 7$ from ① into ②.
$x^2 + 14x + 45 = 3(x + 7)$
$x^2 + 14x - 3x + 45 - 21 = 0$
$x^2 + 11x + 24 = 0$
$(x + 3)(x + 8) = 0$
$x = -3$ or $x = -8$
Reject $x = -8$ as it would make the width 0.
So, the solution is $x = -3$. Then, the length is 5 m and the width is 3 m.
The perimeter is 16 m and the area is 15 m^2.

Chapter 8 Practice Test Page 460 Question 10

a) For the blue parabola:
The vertex is at (0, 0), so the equation has the form $y = ax^2$.
The parabola passes through (3, 3), so substitute $x = 3$, $y = 3$.
$3 = a(3)^2$
$a = \frac{1}{3}$

For the green parabola:
The vertex is at (1, 0), so the equation has the form $y = a(x - 1)^2$.
The parabola passes through (3, 2), so substitute $x = 3$, $y = 2$.

$2 = a(3 - 1)^2$

$a = \frac{1}{2}$

A system of equations for the two quadratic functions is
$y = \frac{1}{3}x^2$ and $y = \frac{1}{2}(x - 1)^2$.

b) Substitute from the first equation into the second.

$\frac{1}{3}x^2 = \frac{1}{2}(x - 1)^2$

$2x^2 = 3(x^2 - 2x + 1)$

$0 = x^2 - 6x + 3$

Use the quadratic formula with $a = 1$, $b = -6$, and $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{24}}{2}$$

$x = 3 \pm \sqrt{6}$

$x \approx 5.45$ or $x \approx 0.55$

Substitute in $y = \frac{1}{3}x^2$ to find the corresponding y-values.

When $x = 3 + \sqrt{6}$:

$y = \frac{1}{3}(3 + \sqrt{6})^2$

$y = \frac{1}{3}(9 + 6\sqrt{6} + 6)$

$y = 5 + 2\sqrt{6}$

$y \approx 9.90$

When $x = 3 - \sqrt{6}$:

$y = \frac{1}{3}(3 - \sqrt{6})^2$

$y = \frac{1}{3}(9 - 6\sqrt{6} + 6)$

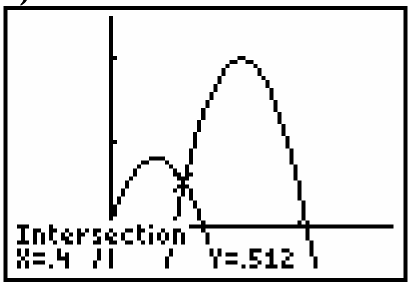
$y = 5 - 2\sqrt{6}$

$y \approx 0.10$

The solutions are approximately (5.45, 9.90) and (0.55, 0.10).

Chapter 8 Practice Test Page 460 Question 11

a)



The solution is (0.4, 0.512).

b) Example: This is the point at which the second part of the jump starts. It is 0.4 cm to the right of and 0.512 cm above the start of the jump.

Chapter 8 Practice Test Page 460 Question 12

Determine the x-coordinates of A and B by finding the x-intercepts of the quadratic.
Solve $0 = -x^2 + 4x + 26.5$ or $x^2 - 4x - 26.5 = 0$
Use the quadratic formula with $a = 1$, $b = -4$, $c = -26.5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-26.5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{122}}{2}$$

$x \approx 7.52$ or $x \approx -3.52$
So the approximate coordinates of A are (–3.52, 0) and of B are (7.52, 0).

Next determine the points of intersection for the line and the parabola. One of the points should be the answer for A.
Substitute $y = 1.5x + 5.25$ into the equation for the parabola.

$$1.5x + 5.25 = -x^2 + 4x + 26.5$$
$$x^2 - 4x + 1.5x - 26.5 + 5.25 = 0$$
$$x^2 - 2.5x - 21.25 = 0$$

Use the quadratic formula with $a = 1$, $b = -2.5$, $c = -21.25$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2.5) \pm \sqrt{(-2.5)^2 - 4(1)(-21.25)}}{2(1)}$$
$$x = \frac{2.5 \pm \sqrt{91.25}}{2}$$
$x \approx 6.03$ or $x \approx -3.52$

So, A is confirmed as approximately (–3.52, 0).

Substitute $x = 6.03$ to determine the y-coordinate of C.
$$y = 1.5\left(\frac{2.5 + \sqrt{91.25}}{2}\right) + 5.25$$
$y \approx 14.29$

AB is the base of ΔABC and is approximately 3.52 + 7.52 or 11.04 units.

The height of the triangle is the y-coordinate of C. The height is about 14.29 units.

Then,

Area = 0.5(base)(height)

Area ≈ 0.5(11.04)(14.29)

Area ≈ 78.88 square units